

## Homework 2

4.9. Suppose 6 out of 15 students in a grade-school class develop influenza, whereas 20% of grade-school students nationwide develop influenza. Is there evidence of an excessive number of cases in the class? That is what is the probability of obtaining at least 6 cases in this case if the nationwide rate holds true?

**Solution.** Here the question is about the number of events and we know the maximum is 15. (There are 15 students). So it is a problem about Binomial Distribution.

Let  $X$  be the number of students out 15 students in a grad-school class develop influenza.  $X$  has Binomial Distribution with  $n=15$  and  $p=.2$  (We define success as the student develops influenza). Then our goal is  $P(X \geq 6)$ .

$$\begin{aligned} P(X \geq 6) &= P(X = 6) + P(X = 7) + \cdots + P(X = 15) \\ &= .043 + .0138 + .0035 + \cdots + .0000 = .611 \end{aligned}$$

So the probability of obtaining at least 6 cases in this case is .611 if the nationwide rate holds true.

4.15. If 500 newborns are screened at the inner-city hospital, then what is the exact binomial probability of at least 5 HIV-positive test results?

**Solution.** Very clearly this is a problem about binomial. Let  $X$  be the number of HIV-positive test results out of 500. Then  $X$  has a binomial distribution with  $n=500$  and  $p=.008$  (We can find  $p$  from Table 4.14, which is on top of page 113). Then our goal is  $P(X \geq 5)$

$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - [P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)] \\ &= 1 - \left[ \binom{500}{0} .008^0 (1 - .008)^{500} + \binom{500}{1} .008^1 (1 - .008)^{499} + \right. \\ &\quad \left. \binom{500}{2} .008^2 (1 - .008)^{498} + \binom{500}{3} .008^3 (1 - .008)^{497} + \right. \end{aligned}$$

$$\begin{aligned} & \binom{500}{4} \cdot .008^4 (1 - .008)^{496} \\ = & 1 - (.0180 + .0727 + .1462 + .1958 + .1962) = .3711 \end{aligned}$$

If some students don't have a powerful enough calculator, you can just leave the formula there.

4.27. Find the probability of not getting any episodes of otitis media in the first year of life.

**Solution.** We already know that the number of episodes per year has a Poisson distribution with parameter  $\lambda = 1.6$ . So let's use  $X$  to denote that number. Then we want  $P(X=0)$ . We can get this very easily using the formula of Poisson probability.

$$P(X = 0) = \frac{e^{-1.6} 1.6^0}{0!} = .202$$

Therefore .202 is the probability of not getting any episodes of otitis media in the first year of life.

4.30. What is the probability that neither sibling will have 3 or more episodes in the first 2 years of life?

**Solution.** In this problem, there are two counts. One is the count of number of siblings, who has 3 or more episodes in two years, the other one is the count of the number of episodes in 2 years for each sibling. Let's use  $Y$  to denote the first count and  $X$  to denote the second count. Then  $Y$  has a binomial distribution with  $n=2$  and success probability  $p$ , which has relationship with  $X$ .  $p = P(X \geq 3)$ , if we define success as the sibling has 3 or more episodes in 2 years. To move on, we should introduce two more variables:  $X_1$  and  $X_2$ . Let  $X_1$  be the number of episodes in the first year of each sibling and  $X_2$  be the number of episodes in the second year. Then, you can see that  $X=X_1+X_2$ . Further, if we assume that  $X_1$  and  $X_2$  are independent, which is kind of reasonable, then  $X$  has a poisson distribution with  $\lambda=1.6+1.6=3.2$ , because  $X_1$  and  $X_2$  both have poisson distribution with  $\lambda=1.6$ . This is the property of poisson distribution and you can find it in one of my handouts. Now we know the distribution of  $X$  and we can work out  $p$ !

$$p = P(X \geq 3) = 1 - P(X \leq 2)$$

$$\begin{aligned}
&= 1 - P(X = 0) - P(X = 1) - P(X = 2) \\
&= 1 - \frac{e^{-3.2}(3.2)^0}{0!} - \frac{e^{-3.2}3.2^1}{1!} - \frac{e^{-3.2}3.2^2}{2!} \\
&= 1 - .0408 - .1304 - .2087 = .62
\end{aligned}$$

$$P(Y = 0) = \binom{2}{0} .62^0 (1 - .62)^2 = .144$$

So, .144 is the probability that neither sibling will have 3 or more episodes in the first 2 years of life.

5.41 If the normal range is 65-120mg/dl, then what percentage of values will fall in the normal range?

**Solution.** We know that values mentioned here have normal distribution with mean 90 mg/dL and standard deviation 38 mg/dL. If we use  $X$  to denote these values then what we want is  $P(65 < X < 120)$ .

$$\begin{aligned}
P(65 < X < 120) &= P(X < 120) - P(X < 65) \\
&= P\left(\frac{X - 90}{38} < \frac{120 - 90}{38}\right) - \\
&\quad P\left(\frac{X - 90}{38} < \frac{65 - 90}{38}\right) \\
&= P(Z < .79) - P(Z < -.66) \\
&= .7852 - P(Z > .66) = .7852 - .2546 = .5306
\end{aligned}$$

So, 53.06% of values will fall in the normal range.

5.42 In some studies only values at least 1.5 times as high as the upper limit of normal are identified as abnormal. What percentage of values would fall in this range?

**Solution.** This problem is asking about  $P(X > 120 \cdot 1.5)$ . This is easy to get.

$$P(X > 120 \cdot 1.5) = P\left(\frac{X - 90}{38} > \frac{180 - 90}{38}\right) = P(Z > 2.37) = .0089.$$

So, only .89% of values would fall in this range.