Inferring Models of cis-Regulatory Modules using Information Theory

BMI/CS 776
www.biostat.wisc.edu/bmi776/
Spring 2019
Colin Dewey
colin.dewey@wisc.edu

Overview

- Biological question
  - What is causing differential gene expression?

- Goal
  - Find regulatory motifs in the DNA sequence

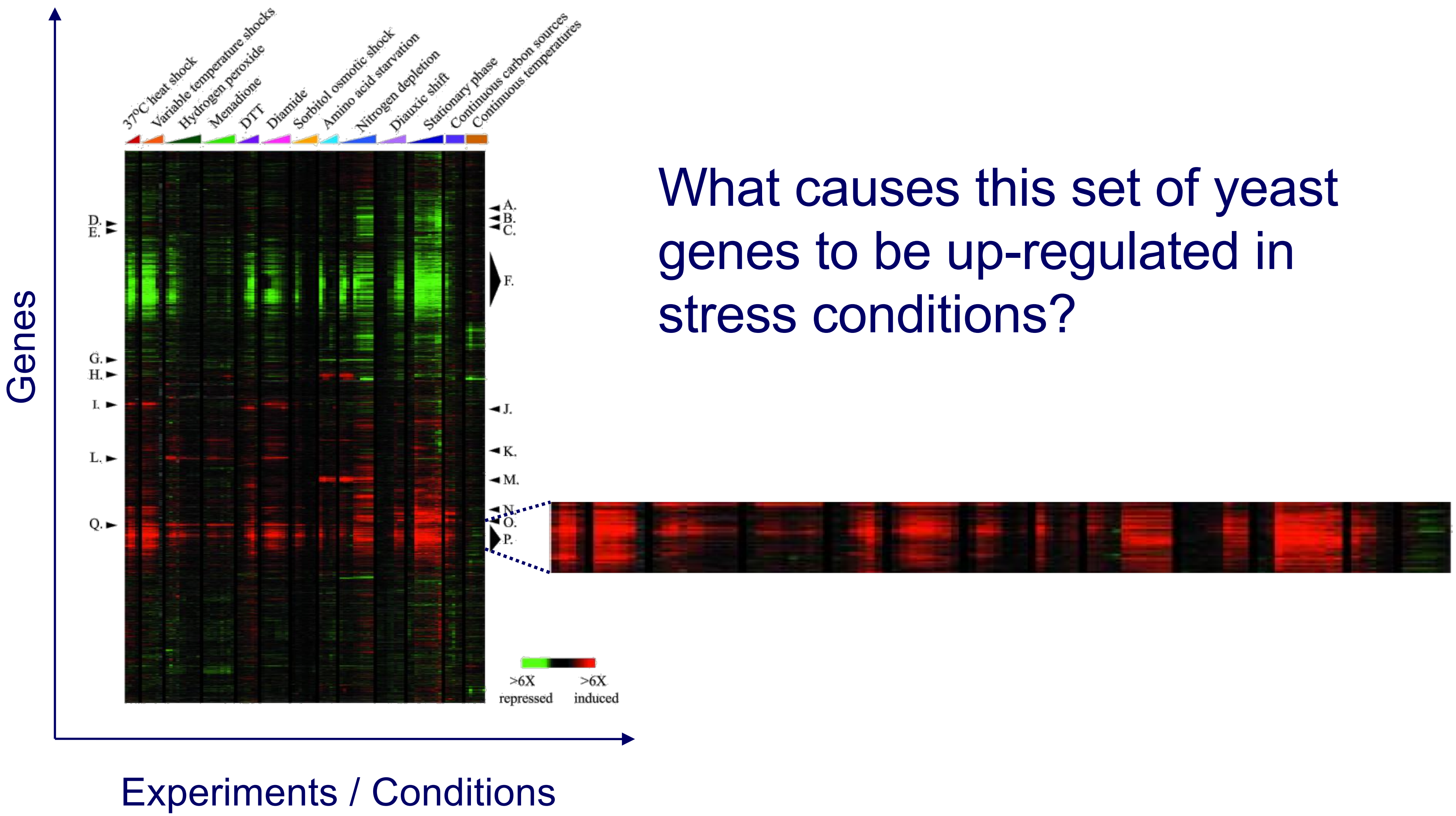
- Solution
  - FIRE (Finding Informative Regulatory Elements)

Goals for Lecture

Key concepts:

- Entropy
- Mutual information (MI)
- Motif logos
- Using MI to identify cis-regulatory module elements

A Common Type of Question

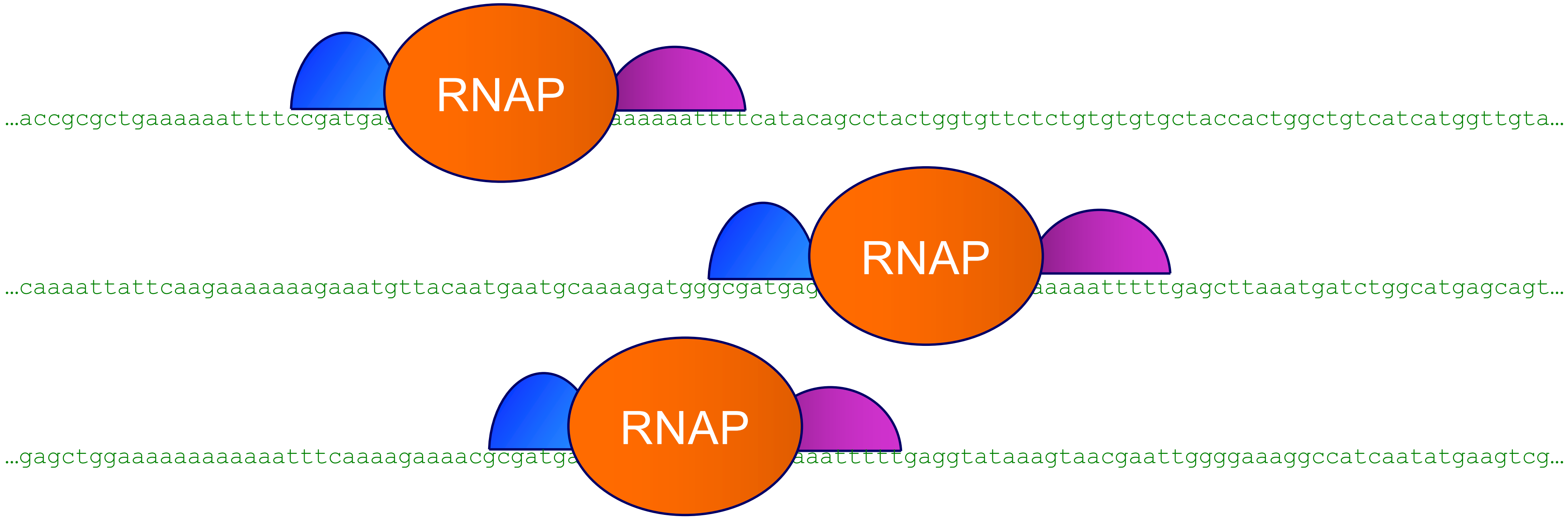


What causes this set of yeast genes to be up-regulated in stress conditions?

Figure from Gasch *et al*., *Mol. Biol. Cell*, 2000

cis-Regulatory Modules (CRMs)

- Co-expressed genes are often controlled by specific configurations of binding sites



Information Theory Background

- Problem
  - Create a code to communicate information
- Example
  - Need to communicate the manufacturer of each bike

Information Theory Background

- Four types of bikes
- Possible code

Type	code
Trek	11
Specialized	10
Cervelo	01
Serotta	00

- Expected number of bits we have to communicate: 2 bits/bike

Information Theory Background

- Can we do better?
- Yes, if the bike types aren't equiprobable

Type, probability	# bits	code
$P(\text{Trek}) = 0.5$	1	1
$P(\text{Specialized}) = 0.25$	2	01
$P(\text{Cervelo}) = 0.125$	3	001
$P(\text{Serotta}) = 0.125$	3	000

- Optimal code uses $-\log_2 P(c)$ bits for event with probability $P(c)$

Information Theory Background

Type, probability	# bits	code
$P(\text{Trek}) = 0.5$	1	1
$P(\text{Specialized}) = 0.25$	2	01
$P(\text{Cervelo}) = 0.125$	3	001
$P(\text{Serotta}) = 0.125$	3	000

- Expected number of bits we have to communicate: 1.75 bits/bike

$$-\sum_{c=1}^{|C|} P(c) \log_2 P(c)$$

Entropy

- Entropy is a measure of uncertainty associated with a random variable

- Can be interpreted as the expected number of bits required to communicate the value of the variable

$$H(C) = -\sum_{c=1}^{|C|} P(c) \log_2 P(c)$$

entropy function for binary variable

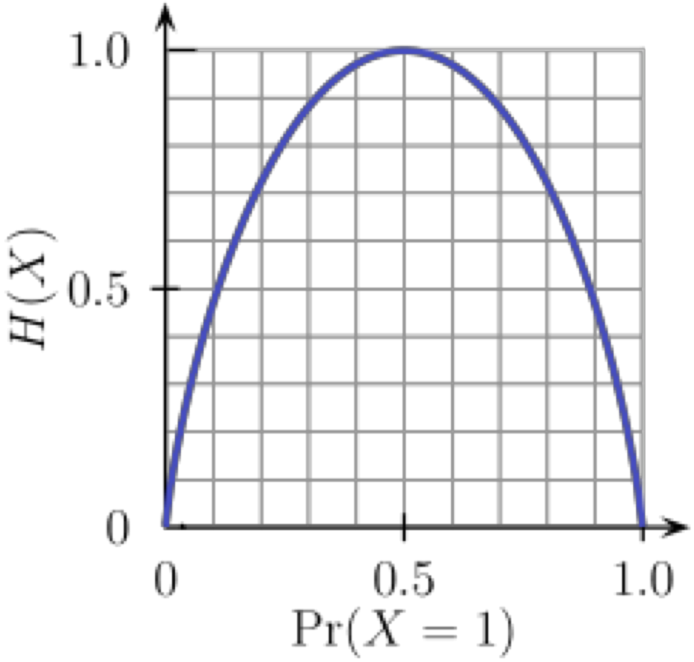
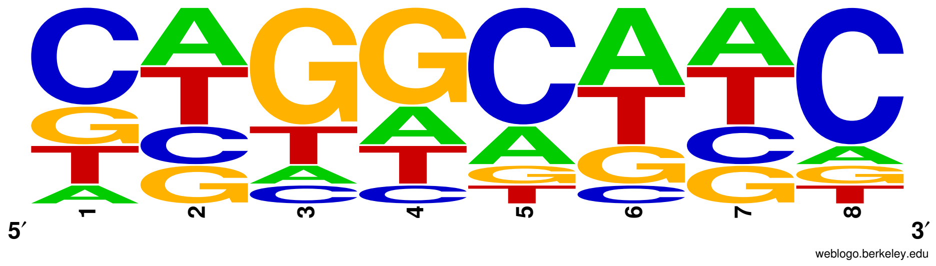


Image from Wikipedia

How is entropy related to DNA sequences?

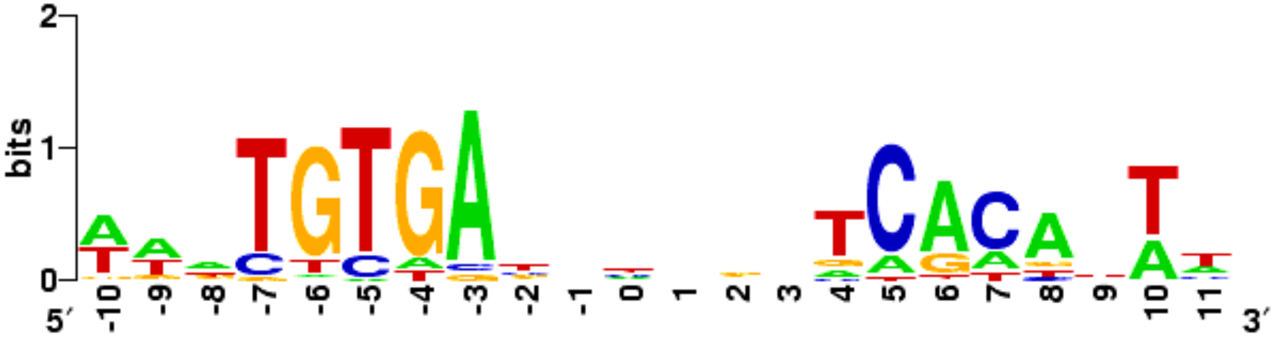
Sequence Logos



- Typically represent a binding site

- Height of each character c is proportional to $P(c)$

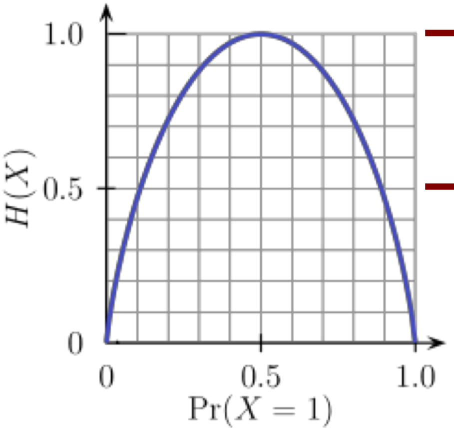
Sequence Logos



- Height of <u>logo</u> at a given position determined by decrease in entropy (from maximum possible)

$$H_{\max} - H(C) = \log_2 N - \left(-\sum_{c} P(c) \log_2 P(c) \right)$$

# of characters in alphabet



decrease in entropy

Mutual Information

- *Mutual information* quantifies how much knowing the value of one variable tells about the value of another

entropy of M

entropy of M conditioned on C

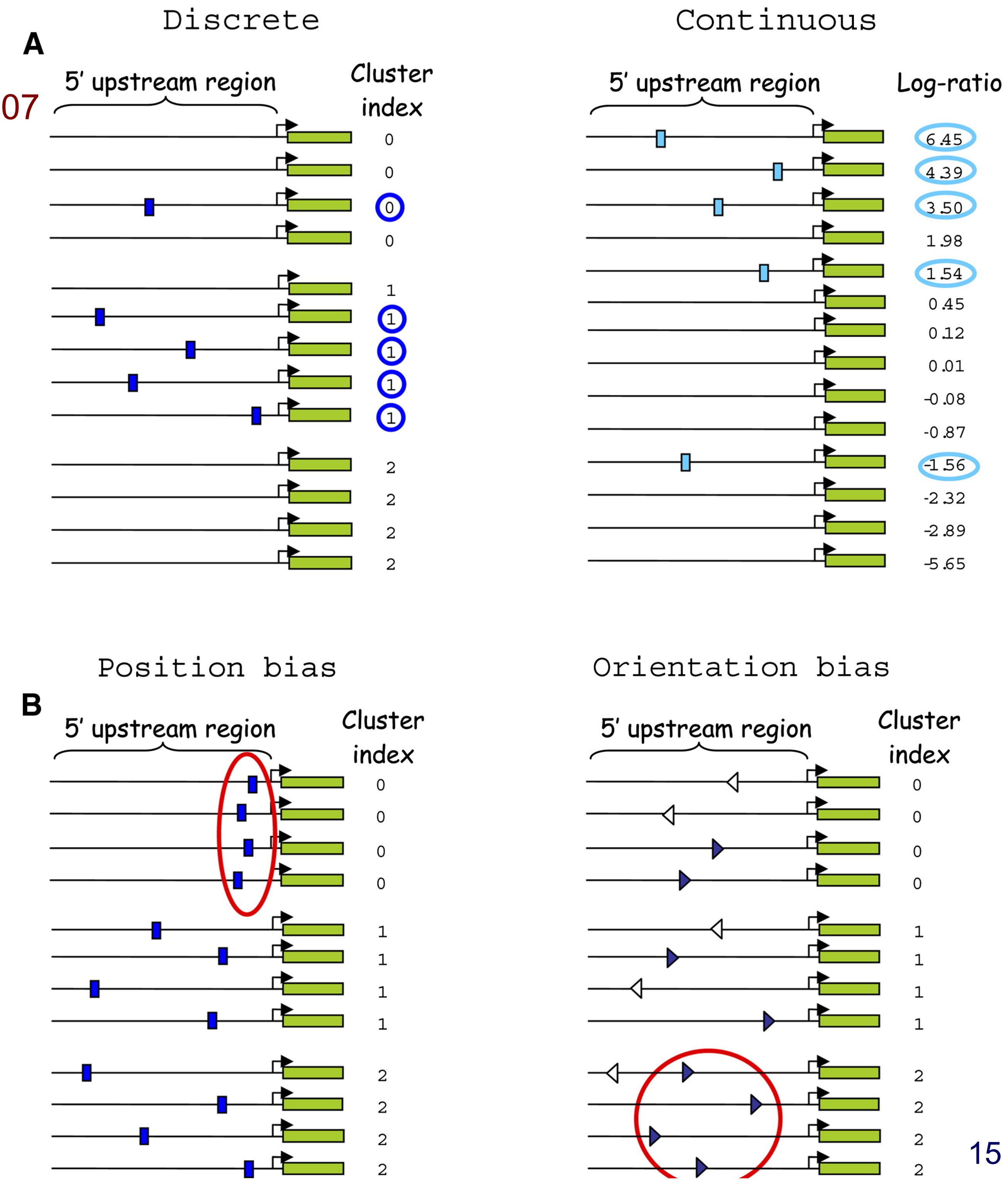
$$I(M;C) = H(M) - H(M \mid C)$$

$$= \sum_{m} \sum_{c} P(m,c) \log_2 \left(\frac{P(m,c)}{P(m)P(c)} \right)$$

FIRE

Elemento et al., *Molecular Cell* 2007

- **F**inding **I**nformative **R**egulatory **E**lements (FIRE)
- **Given** a set of sequences grouped into clusters
- **Find** motifs, and relationships, that have high *mutual information* with the clusters
- Applicable when sequences have continuous values instead of cluster labels



Mutual Information in FIRE

- We can compute the mutual information between a motif and the clusters as follows

$$I(M;C)=\sum_{m=0}^{1}\sum_{c=1}^{|C|}P(m,c)\log_2\frac{P(m,c)}{P(m)P(c)}$$

$m=0,1$ represent absence/presence of motif

c ranges over the cluster labels

Finding Motifs in FIRE

- Motifs are represented by regular expressions; initially each motif is represented by a strict *k*-mer (e.g. TCCGTAC)

1. Test all *k*-mers (*k*=7 by default) to see which have significant mutual information with the cluster label

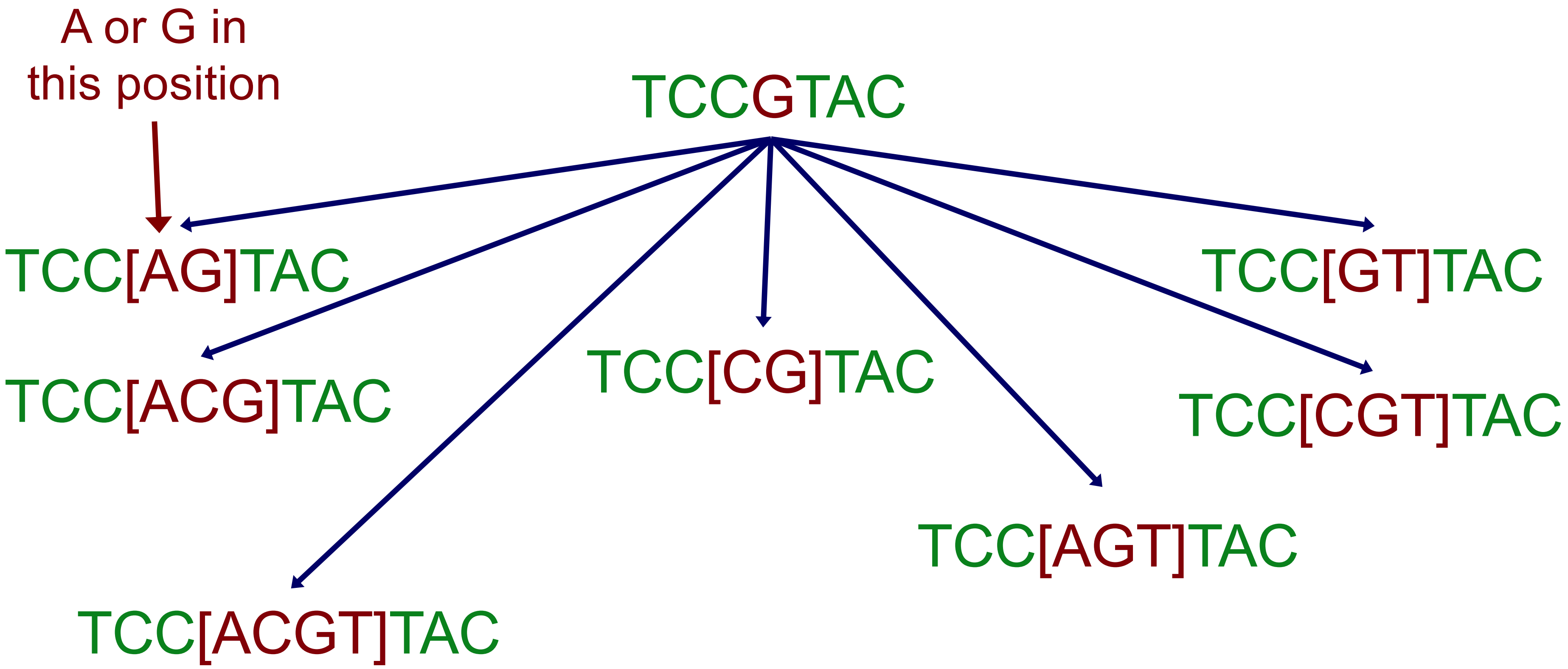
2. Filter *k*-mers using a significance test to obtain motif seeds

3. Generalize each motif seed

4. Filter motifs using a significance test

Key Step in Generalizing a Motif in FIRE

- Randomly pick a position in the motif
- Generalize in all ways consistent with current value at position
- Score each by computing mutual information
- Retain the best generalization



A or G in
this position

TCCGTAC

TCC[AG]TAC

TCC[ACG]TAC

TCC[ACGT]TAC

TCC[CG]TAC

TCC[AGT]TAC

TCC[GT]TAC

TCC[CGT]TAC

Generalizing a Motif in FIRE

```
given: k-mer, n

best ← null
repeat n times
    motif ← k-mer
    repeat
        motif ← GeneralizePosition(motif)    // shown on previous slide
    until convergence (no improvement at any position)
    if score(motif) > score(best)
        best ← motif

return: best
```

Generalizing a Motif in FIRE: Example

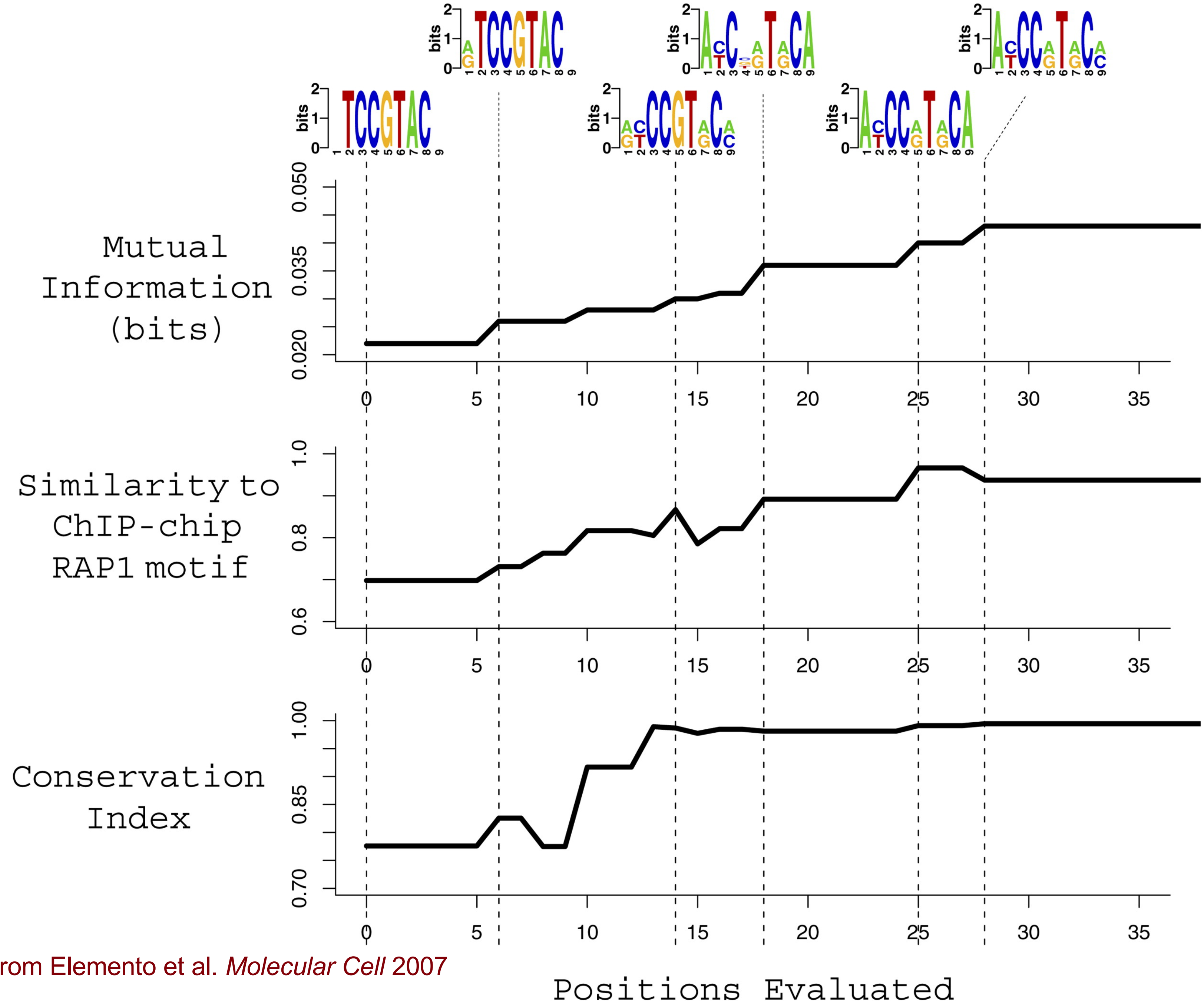


Figure from Elemento et al. *Molecular Cell* 2007

Avoiding Redundant Motifs

- Different seeds could converge to similar motifs

TCCGTAC

TCC[CG]TAC

TCCCTAC

TCC[CG]TAC

- Use mutual information to test whether new motif is unique and contributes new information

$$\frac{I(M;C\,|\,M')}{I(M;M')} > r$$

M'	previous motif
M	new candidate motif
C	expression clusters

Characterizing Predicted Motifs in FIRE

- Mutual information is also used to assess various properties of found motifs
  - orientation bias
  - position bias
  - interaction with another motif

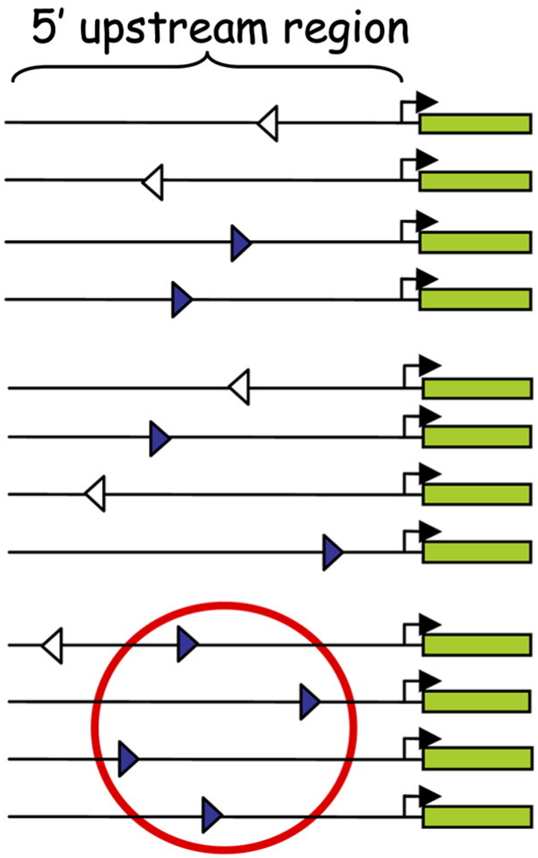
Using MI to Determine Orientation Bias

$$I(S;C)$$

C indicates cluster

$S=1$ indicates motif present on transcribed strand

$S=0$ otherwise (not present or not on transcribed strand)



C	S
0	0
0	0
0	1
0	1
1	0
1	1
1	0
1	1
2	1
2	1
2	1
2	1

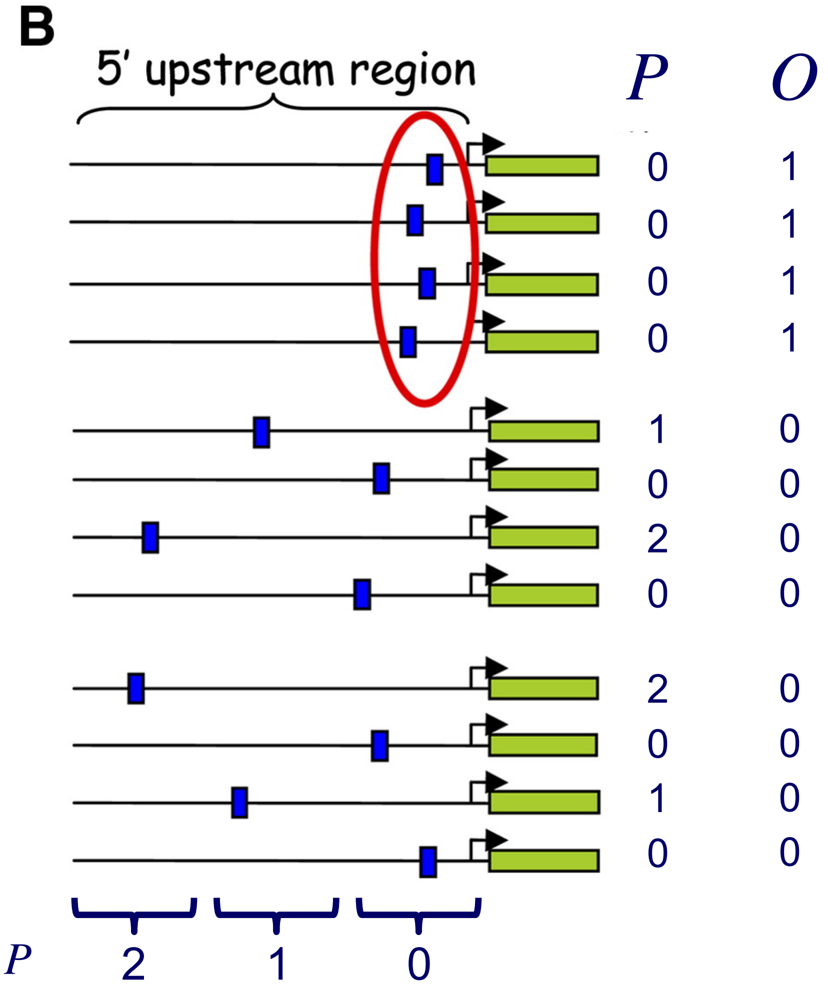
Also compute MI where $S=1$ indicates motif present on complementary strand

Using MI to Determine Position Bias

$$I(P;O)$$

P ranges over position bins

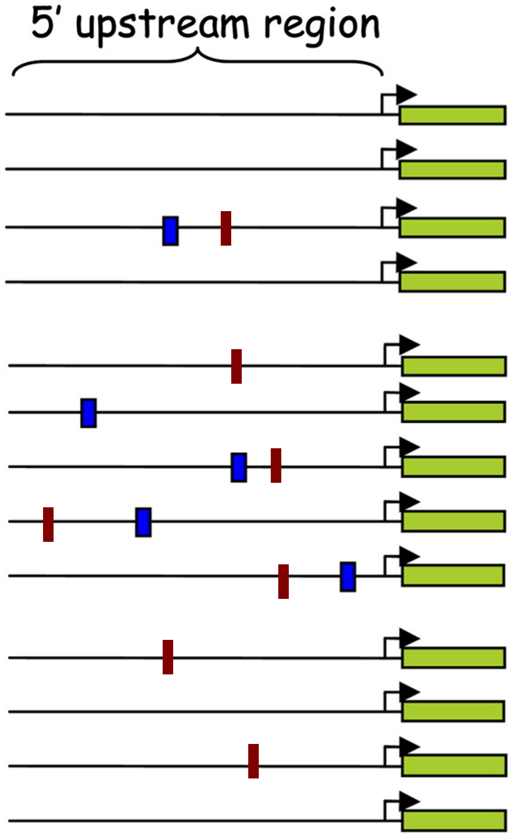
O=0, 1 indicates whether or not the motif is over-represented in a sequence’s cluster



Only sequences containing the motif are considered for this calculation

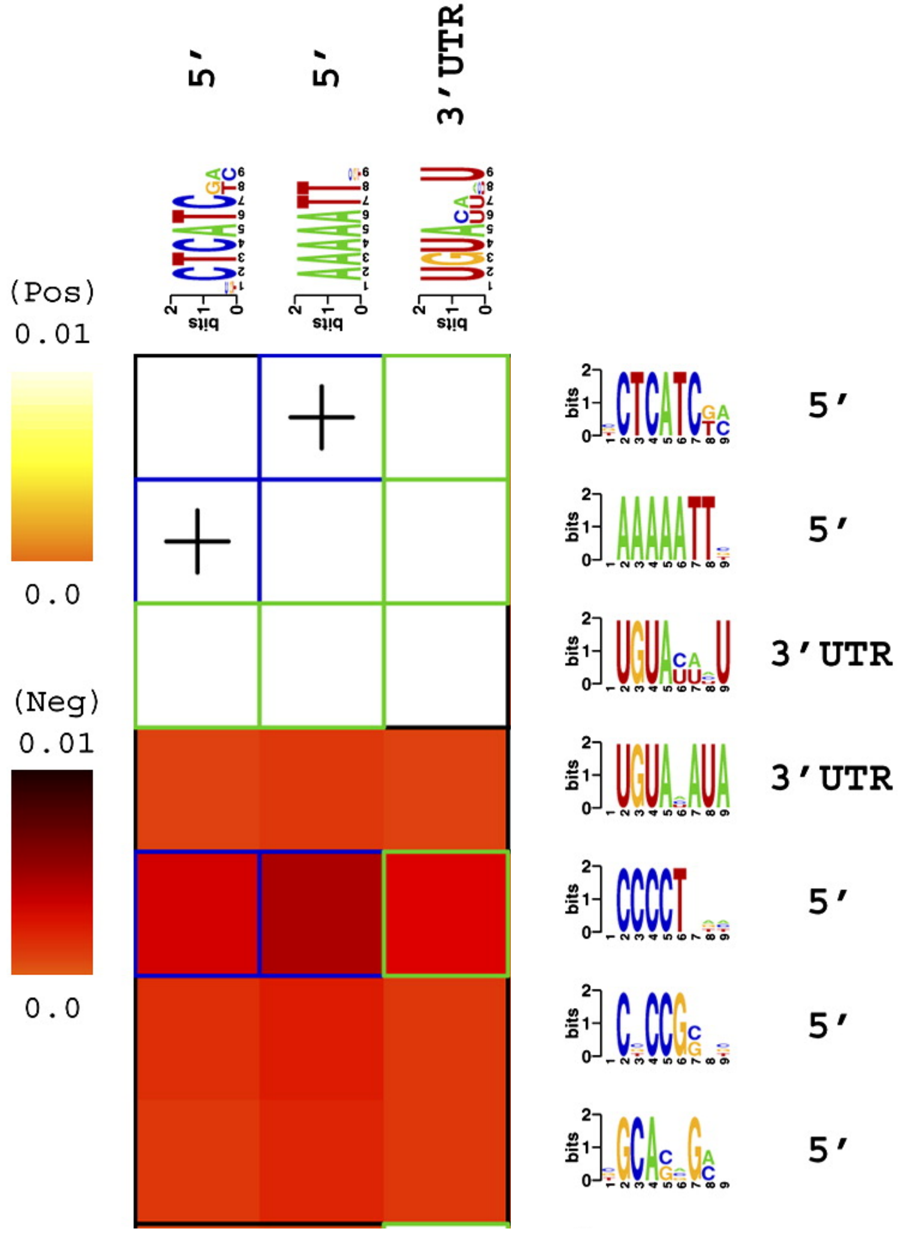
Using MI to Determine Motif Interactions

$I(M_1; M_2)$ $M_1 = 0, 1$ indicates whether or not a sequence has the motif **and** is in a cluster for which the motif is over-represented; similarly for M_2



M_1	M_2
0	0
0	0
0	0
0	0
0	1
1	0
1	1
1	1
1	1
0	1
0	0
0	1
0	0

Motif Interactions Example



Yeast motif-motif interactions

White: positive association

Dark red: negative association

Blue box: DNA-DNA

Green box: DNA-RNA

Plus: spatial co-localization

Discussion of FIRE

- FIRE
  - mutual information used to identify motifs and relationships among them
  - motif search is based on generalizing informative *k*-mers

- Consider advantages and disadvantages of *k*-mers versus PWMs

- In contrast to many motif-finding approaches, FIRE takes advantage of *negative* sequences

- FIRE returns all informative motifs found