**When Wilcoxon-Mann-Whitney meets non-compliance**

**Abstract:** The nonparametric, rank-based Wilcoxon-Mann-Whitney (WMW) test is a popular alternative to the standard $t$-test for its robustness and potential efficiency. In addition, the WMW test statistic also provides an easily interpretable measure of effect size, in terms of probabilistic shift. Cast in the Rubin Causal Model, the probabilistic (shift) index has received considerable attention under randomized trials and under non-randomized trials with measured confounders. While non-compliance with unknown confounders is a common presence in randomized trials, little is known about the proper use or adaptation of the WMW statistics in such cases. Following the seminal work of Angrist, Imbens, and Rubin (1996) on the local average treatment effect on the compliers, we seek to establish a parallel, instrumental-variable-based framework for WMW statistics with an insistence on causal interpretability. Three questions are of interest:

1. In the presence of non-compliance, what constitutes a local probabilistic index on the compliers?
2. How do we construct an estimator for the local probabilistic index?
3. How does the test based on this estimator compare with the standard intent-to-treat WMW test in terms of power?

We will show that these questions can be answered by using the idea of principal stratification along with the help of von Mises calculus. The resultant procedures are applied to a job training study as an illustration.