Evaluating Machine-Learning Methods (Part 2)

Mark Craven and David Page Computer Sciences 760 Spring 2018

www.biostat.wisc.edu/~craven/cs760/

Some of the slides in these lectures have been adapted/borrowed from materials developed by Tom Dietterich, Pedro Domingos, Tom Mitchell, David Page, and Jude Shavlik

Goals for the lecture

you should understand the following concepts

- · confidence intervals for error
- pairwise t-tests for comparing learning systems
- · scatter plots for comparing learning systems
- · lesion studies
- model selection
- · validation (tuning) sets
- · internal cross validation

Confidence intervals on error

Given the observed error (accuracy) of a model over a limited sample of data, how well does this error characterize its accuracy over additional instances?

Suppose we have

- a learned model h
- a test set S containing n instances drawn independently of one another and independent of h
- *n* ≥ 30
- h makes r errors over the n instances

our best estimate of the error of h is

$$error_{S}(h) = \frac{r}{n}$$

Confidence intervals on error

With approximately C% probability, the true error lies in the interval

$$error_{S}(h) \pm z_{C} \sqrt{\frac{error_{S}(h)(1 - error_{S}(h))}{n}}$$

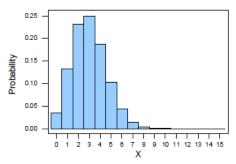
where z_{C} is a constant that depends on C (e.g. for 95% confidence, z_{C} =1.96)

Confidence intervals on error

How did we get this?

1. Our estimate of the error follows a binomial distribution given by n and p (the true error rate over the data distribution)

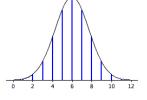
Binomial distribution with n = 15 and p = 0.2



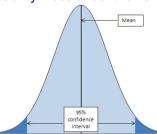
2. Most common way to determine a binomial confidence interval is to use the *normal approximation* (although can calculate exact intervals if *n* is not too large)

Confidence intervals on error

2. When $n \ge 30$, and p is not too extreme, the normal distribution is a good approximation to the binomial



3. We can determine the C% confidence interval by determining what bounds contain C% of the probability mass under the normal



Alternative approach: confidence intervals using bootstrapping

- bootstrap sample: given n examples in data set, randomly, uniformly, independently draw n examples with replacement
- repeat 1000 (or 10,000) times:
 - · draw bootstrap sample
 - · measure error on bootstrap sample
 - for 95% confidence interval, lower (upper) bound is set such that 2.5% of runs yield lower (higher) error

Comparing learning systems

How can we determine if one learning system provides better performance than another

- for a particular task?
- across a set of tasks / data sets?

Motivating example

Accuracies on test sets

System A: 80% 50 75 ... 99
System B: 79 49 74 ... 98
δ: +1 +1 +1 ... +1

- Mean accuracy for System A is better, but the standard deviations for the two clearly overlap
- Notice that System A is always better than System B

Comparing systems using a paired t test

- consider δ's as observed values of a set of i.i.d. random variables
- null hypothesis: the 2 learning systems have the same accuracy
- alternative hypothesis: one of the systems is more accurate than the other
- hypothesis test:
 - use paired *t*-test do determine probability *p* that mean of δ's would arise from null hypothesis
 - if p is sufficiently small (typically < 0.05) then reject the null hypothesis

Comparing systems using a paired t test

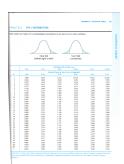
1. calculate the sample mean

$$\overline{\delta} = \frac{1}{n} \sum_{i=1}^{n} \delta_{i}$$

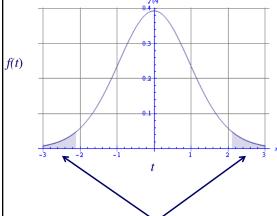
2. calculate the *t* statistic

$$t = \frac{\overline{\delta}}{\sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (\delta_i - \overline{\delta})^2}}$$

3. determine the corresponding *p*-value, by looking up *t* in a table of values for the Student's *t*-distribution with *n-1* degrees of freedom







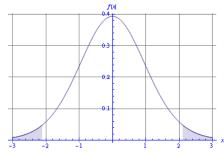
The null distribution of our *t* statistic looks like this

The p-value indicates how far out in a tail our t statistic is

If the *p*-value is sufficiently small, we reject the <u>null</u> hypothesis, since it is unlikely we'd get such a *t* by chance

for a two-tailed test, the *p*-value represents the probability mass in these two regions

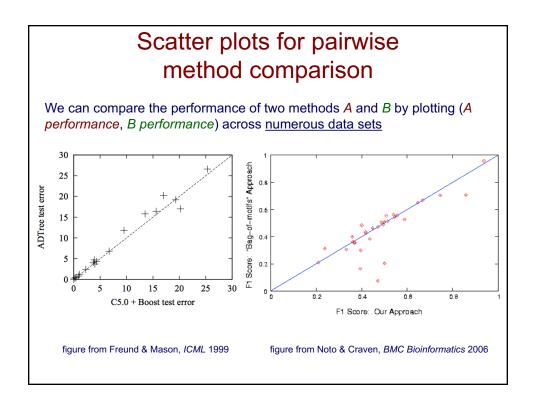
Why do we use a two-tailed test?

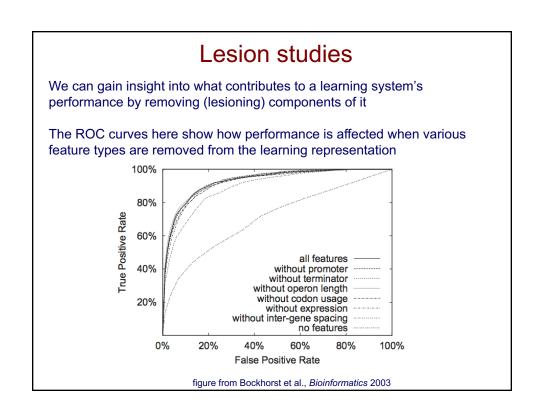


- a two-tailed test asks the question: is the accuracy of the two systems different
- a one-tailed test asks the question: is system A better than system B
- a priori, we don't know which learning system will be more accurate (if there is a difference) – we want to allow that either one might be

Comments on hypothesis testing to compare learning systems

- the paired t-test can be used to compare two <u>learning</u> <u>systems</u>
- other tests (e.g. McNemar's χ^2 test) can be used to compare two learned models
- a statistically significant difference is not necessarily a large-magnitude difference





To avoid pitfalls, ask

- 1. Is my held-aside test data really representative of going out to collect new data?
 - Even if your methodology is fine, someone may have collected features for positive examples differently than for negatives – should be randomized
 - Example: samples from cancer processed by different people or on different days than samples for normal controls

To avoid pitfalls, ask

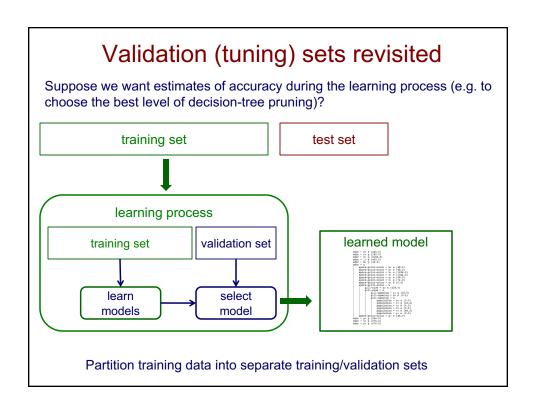
- 2. Did I repeat my entire data processing procedure on every fold of cross-validation, using only the training data for that fold?
 - On each fold of cross-validation, did I ever access in any way the label of a test instance?
 - Any preprocessing done over entire data set (feature selection, parameter tuning, threshold selection) must not use labels

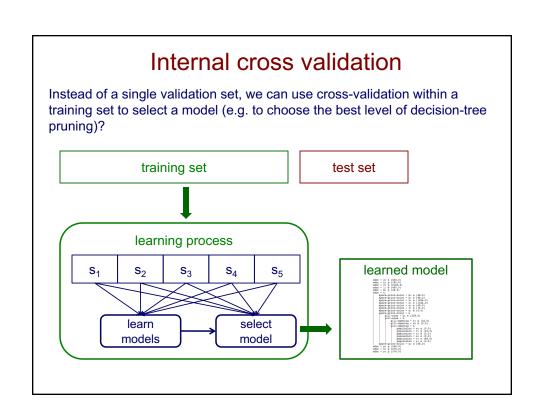
To avoid pitfalls, ask

- 3. Have I modified my algorithm so many times, or tried so many approaches, on this same data set that I (the human) am overfitting it?
 - Have I continually modified my preprocessing or learning algorithm until I got some improvement on this data set?
 - If so, I really need to get some additional data now to at least test on

Model selection

- model selection is the task of selecting a model from a set of candidate models
 - selecting among decision trees with various levels of pruning
 - selecting k in k-NN
 - etc.
- one approach to model selection is to use a tuning set or internal cross validation





Example: using internal cross validation to select *k* in *k*-NN

given a training set

- 1. partition training set into n folds, $s_1 \dots s_n$
- 2. for each value of *k* considered

for i = 1 to n

learn k-NN model using all folds but s_i evaluate accuracy on s_i

- 3. select k that resulted in best accuracy for $s_1 \dots s_n$
- 4. learn model using entire training set and selected *k*

the steps inside the box are run independently for each training set (i.e. if we're using 10-fold CV to measure the overall accuracy of our *k*-NN approach, then the box would be executed 10 times)