

HotNet

# Background

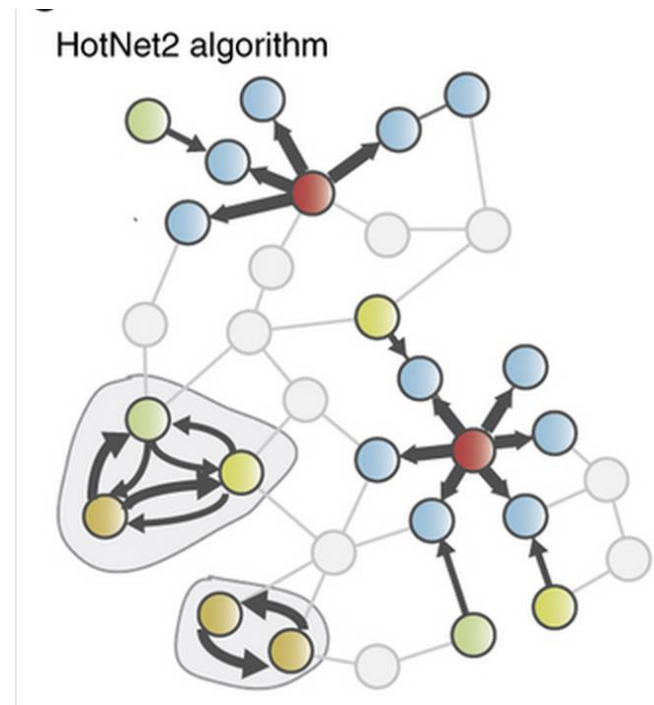
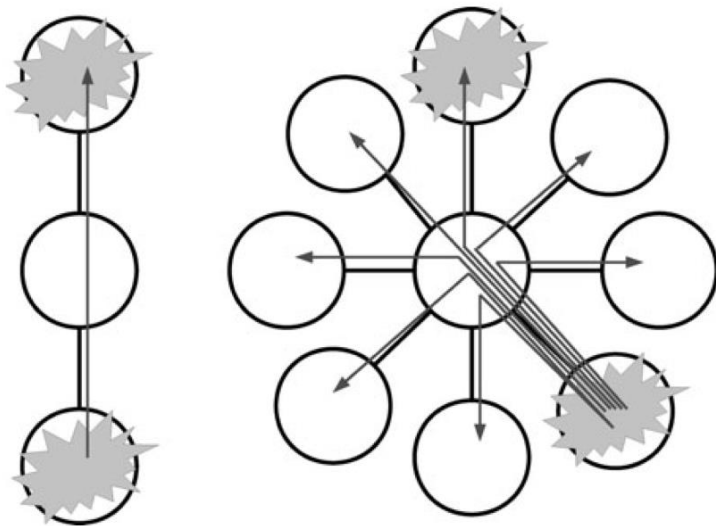
- Determine significantly mutated subnetworks in a large gene interaction network
- Problems with current methods
  - Frequency doesn't always predict significance
  - Naïve subnetwork analysis
    - Enumeration prohibits subnetworks of reasonable size
    - Large number of hypotheses makes statistically significant difficult
    - Hub genes make for small gene diameters

# HotNet Overview

1. Formulate an influence measure between pairs of genes in the network
2. Identify subnetworks with Combinatorial Model or Enhanced Influence Model
3. Two-stage multiple hypothesis test to mitigate testing of large number of hypotheses

# Influence Graph

- Identify subnetworks that are significant with respect to a set of mutated genes



# Diffusion

$f_v^s(t)$  Amount of fluid @ node  $V$  at time  $T$

$\mathbf{f}^s(t) = [f_1^s(t), \dots, f_n^s(t)]^T$  Amount of fluid at all nodes

$L_\gamma = L + \gamma I$   $L$  is the laplacian matrix of the graph

$\frac{d\mathbf{f}^s(t)}{dt} = -L_\gamma \mathbf{f}^s(t) + \mathbf{b}^s u(t)$ , Dynamics of the continuous process

- Interpret  $f_i$  as the influence of gene  $g_s$  on  $g_i$

# Combinatorial Model

- Takes in influence measure between genes to discover significant subnetworks

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## Combinatorial Algorithm

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**Input:** Influence graph  $G_I$  and parameters  $\delta$  and  $k$

**Output:** Connected subgraph  $\mathcal{C}$  of  $G_I(\delta)$  with  $k$  vertices

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1 Construct  $G_I(\delta)$  by removing from  $G_I$  all edges with weight  $< \delta$ ;  
2  $\mathcal{C} \leftarrow \emptyset$ ;  
3 for each node  $v \in V$  do  
4    $\mathcal{C}_v \leftarrow \{v\}$ ;  
5   for each  $u \in V \setminus \{v\}$  do  $p_v(u) \leftarrow$  shortest path from  $v$  to  $u$  in  $G_I(\delta)$ ;  
6   while  $|\mathcal{C}_v| < k$  do  
     //  $\ell_v(u)$  = set of nodes in  $p_v(u)$ ;  $P_v(u)$  = elements of  $I$  covered by  
      $\ell_v(u)$ ;  $P_{\mathcal{C}_v}$  = elements covered by  $\mathcal{C}_v$ ;  $P_{\mathcal{C}}$  = elements covered by  $\mathcal{C}$   
7      $u \leftarrow \arg \max_{u \in V \setminus \mathcal{C}_v: |\ell_v(u) \cup \mathcal{C}_v| \leq k} \left\{ \frac{|P_v(u) \setminus P_{\mathcal{C}_v}|}{|\ell_v(u) \setminus \mathcal{C}_v|} \right\}$ ;  
8      $\mathcal{C}_v \leftarrow \ell_v(u) \cup \mathcal{C}_v$ ;  
9   if  $|P_{\mathcal{C}_v}| > |P_{\mathcal{C}}|$  then  $\mathcal{C} \leftarrow \mathcal{C}_v$ ;  
10 return  $\mathcal{C}$ ;
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# Enhanced Influence Model

- Enhance the influence measure between genes by the number of mutations observed in each gene

## Enhanced Influence Algorithm

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**Input:** Influence graph  $G_I$  and parameter  $\delta$

**Output:** Connected components of  $H(\delta)$

- 1  $V_H \leftarrow \{g_j : \mathcal{S}_j \neq \emptyset\};$
  - 2  $E \leftarrow \{g_j, g_k : g_j, g_k \in V_H, g_j \neq g_k\};$
  - 3  $H \leftarrow (V_H, E, h);$
  - 4  $E(\delta) \leftarrow \{(g_j, g_k) \in E : h(g_j, g_k) \geq \delta\};$
  - 5  $H(\delta) \leftarrow (V_H, E(\delta));$
  - 6 **return** connected components of  $H(\delta);$
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# Statistics

- Calibrates with  $H_0^{\text{sample}}$  and  $H_0^{\text{gene}}$ 
  - Sample: mutations placed at random nodes
  - Gene: Move genes around.....?
- Compute significance of **number** of subnetworks
- Bound FDR

# Experimental Data

TABLE 1. RESULTS OF THE COMBINATORIAL MODEL

<i>Dataset</i>	<i>k</i>	<i>Samples</i>	<i>p-value</i>		<i>Pathway enrichment p-value</i>		
			$H_0^{\text{sample}}$	$H_0^{\text{gene}}$	<i>All</i>	<i>RTK/RAS/PI(3)K</i>	<i>p53</i>
GBM	10	67	$<10^{-10}$	$4 \times 10^{-3}$	$3 \times 10^{-4}$	$8 \times 10^{-4}$	0.19
	20	78	$<10^{-10}$	$<10^{-3}$	$10^{-5}$	$8 \times 10^{-5}$	0.05
Lung	10	140	$<10^{-10}$	0.02	$8 \times 10^{-6}$	/	
	20	151	$<10^{-10}$	0.03	$3 \times 10^{-3}$	/	

$k$  is the number of genes in the subnetwork. *Samples* is the number of samples in which the subnetwork is mutated. *p-value* is the probability of observing a connected subgraph of size  $k$  mutated in a number of samples  $\geq \text{samples}$  under the random model  $H_0^{\text{sample}}$  or  $H_0^{\text{gene}}$ . *enrichment p-value* is the  $p$ -value of the hypergeometric test for overlap between genes in the identified subgraph and genes reported significant pathways in TCGA (2008) or Ding et al. (2008). For GBM, *enrichment p-value* is the  $p$ -value of the hypergeometric test for RTK/RAS/PI(3)K and p53 pathways.

