

# QTL Mapping II:

## Introduction

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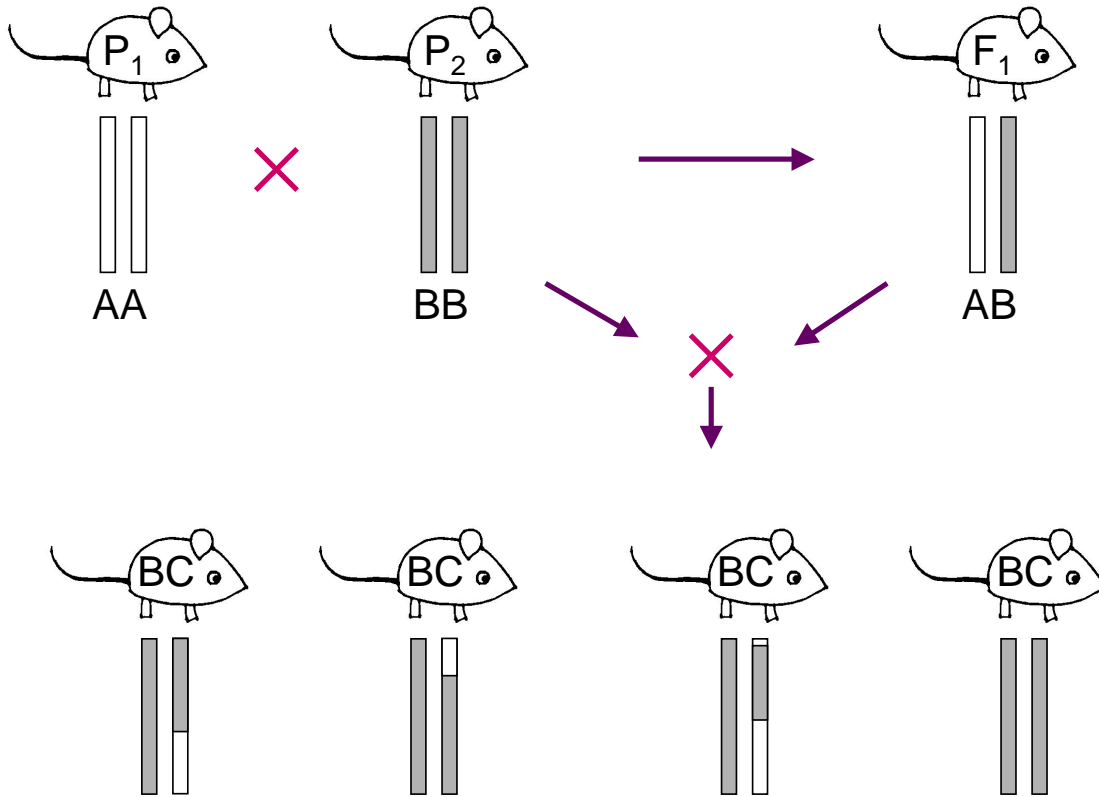
## Outline

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- Experiments and data
- Goals, statistical structure
- Models
- ANOVA, interval mapping
- LOD thresholds
- CIs for QTL location
- Selection bias
- Multiple QTLs
- Likelihood vs. Bayes
- Model selection
- Epistasis
- The X chromosome
- Selective genotyping
- Covariates
- Non-normal traits
- The need for good data

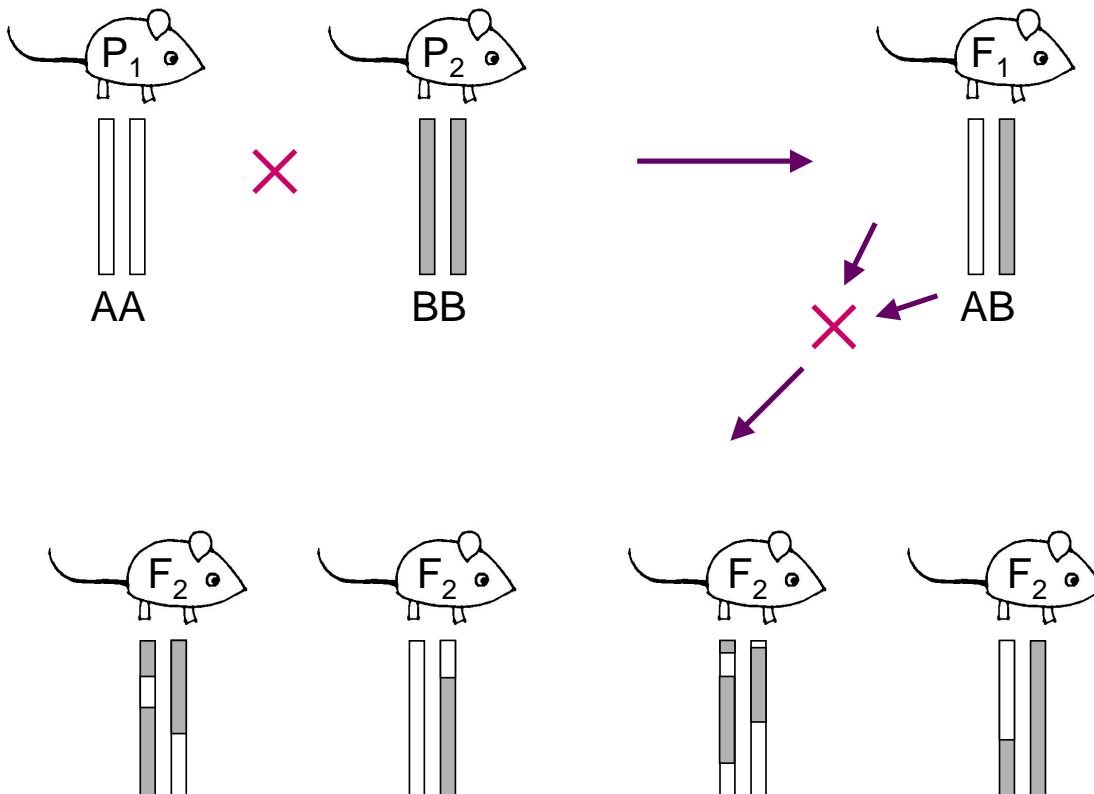
## Backcross experiment

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## Intercross experiment

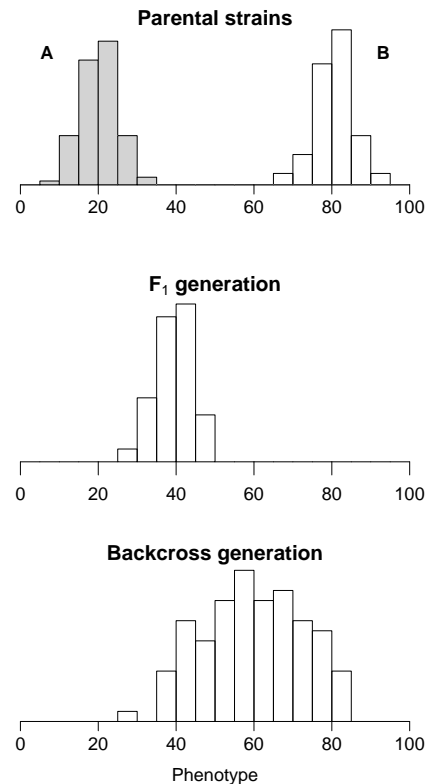
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# Phenotype distributions

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- Within each of the parental and F<sub>1</sub> strains, individuals are genetically identical.
- Environmental variation may or may not be constant with genotype.
- The backcross generation exhibits genetic as well as environmental variation.



## Data

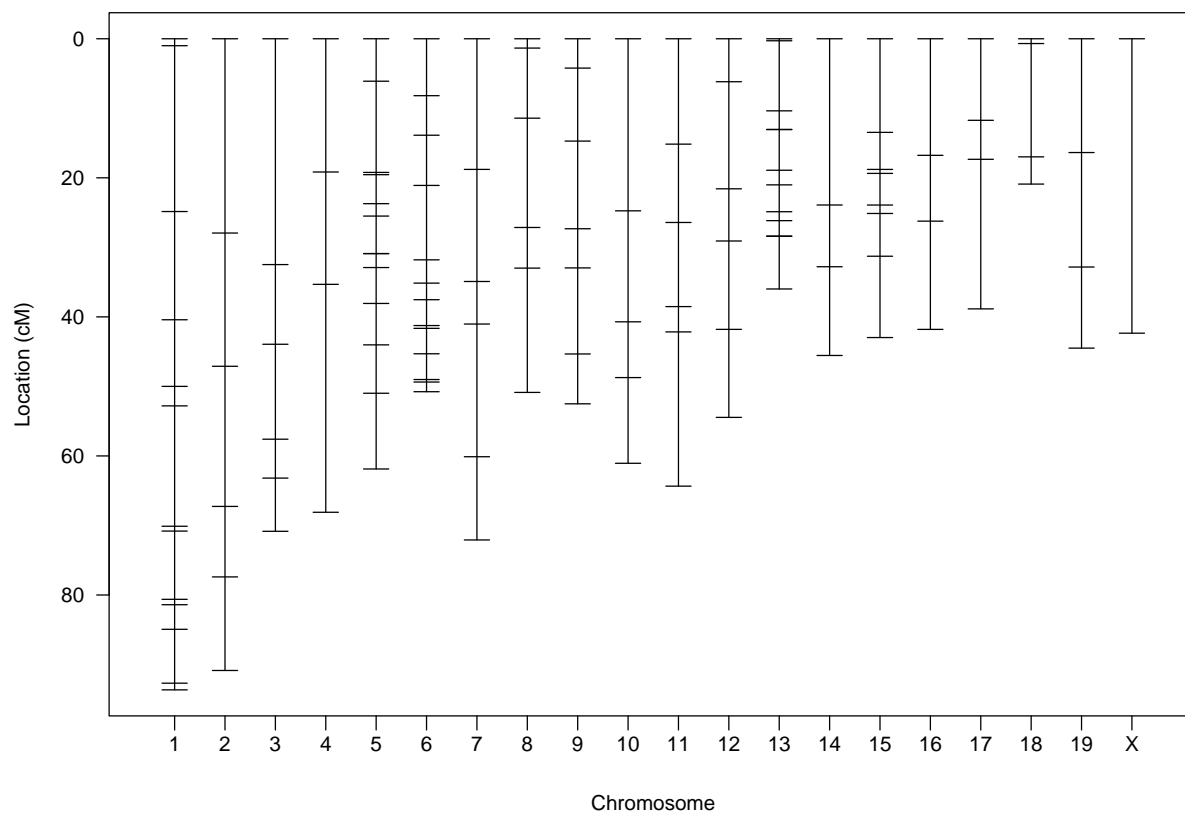
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**Phenotypes:**  $y_i$  = trait value for individual  $i$

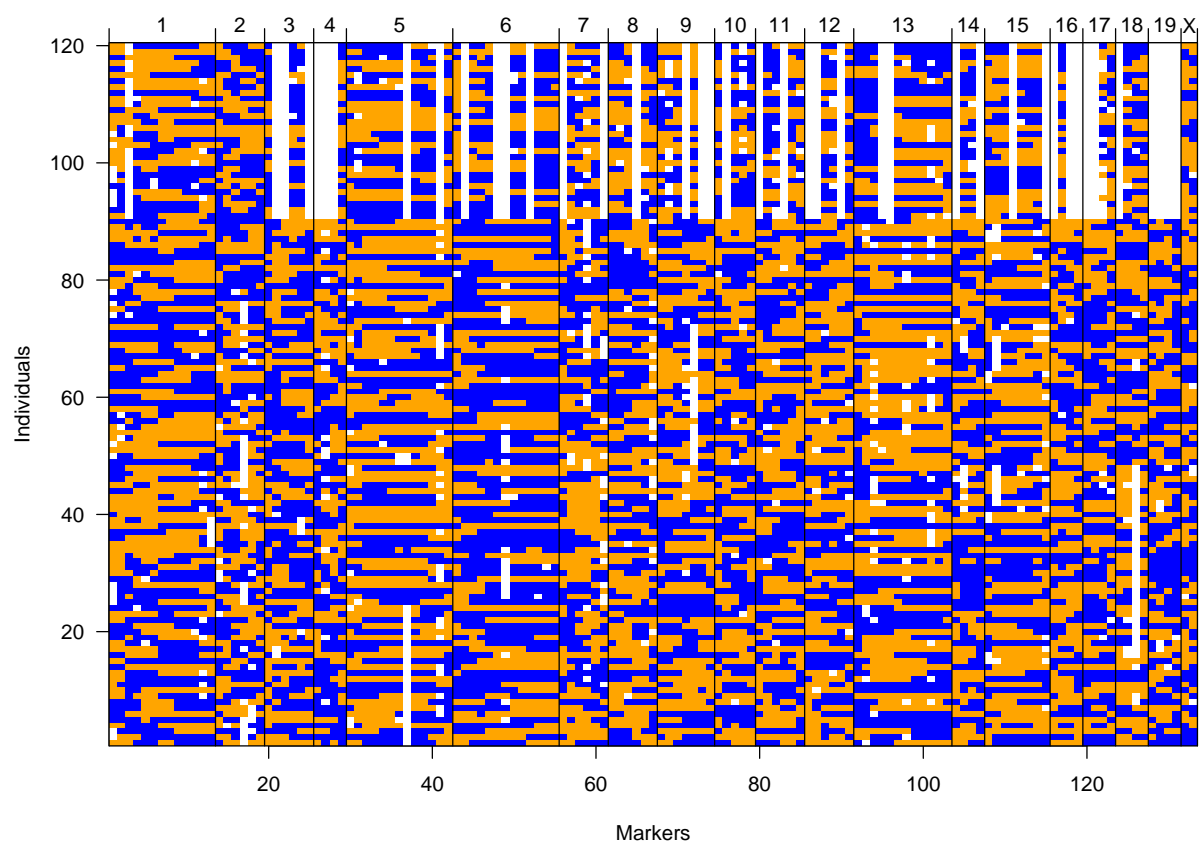
**Genotypes:**  $x_{ij}$  = 0/1 if mouse  $i$  is BB/AB at marker  $j$   
(or 0/1/2, in an intercross)

**Genetic map:** Locations of markers

## Genetic map



## Genotype data



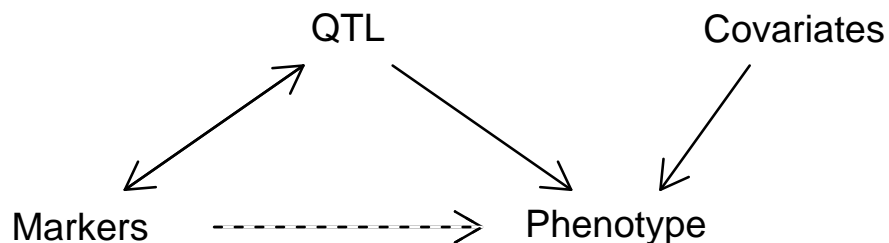
# Goals

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- Detect QTLs (and interactions between QTLs)
- Confidence intervals for QTL location
- Estimate QTL effects (effects of allelic substitution)

## Statistical structure

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The missing data problem:

Markers  $\longleftrightarrow$  QTL

The model selection problem:

QTL, covariates  $\longrightarrow$  phenotype

# Models: Recombination

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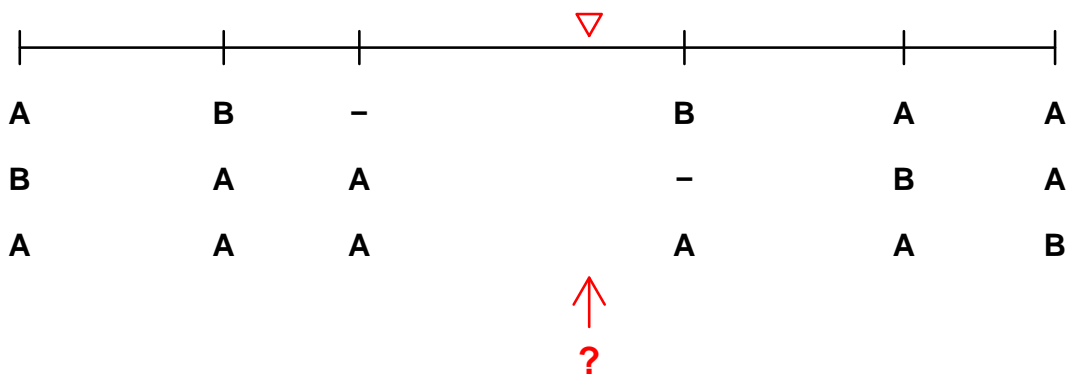
We assume no crossover interference.

⇒ Points of exchange (crossovers) are according to a **Poisson process**.

⇒ The  $\{x_{ij}\}$  (marker genotypes) form a **Markov chain**

## Example

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## Models: Genotype $\longleftrightarrow$ Phenotype

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Let  $y$  = phenotype  
 $g$  = whole genome genotype

Imagine a small number of QTLs with genotypes  $g_1, \dots, g_p$ .  
( $2^p$  distinct genotypes)

$$E(y|g) = \mu_{g_1, \dots, g_p} \quad \text{var}(y|g) = \sigma_{g_1, \dots, g_p}^2$$

## Models: Genotype $\longleftrightarrow$ Phenotype

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**Homoscedasticity** (constant variance):  $\sigma_g^2 \equiv \sigma^2$

**Normally distributed residual variation:**  $y|g \sim N(\mu_g, \sigma^2)$ .

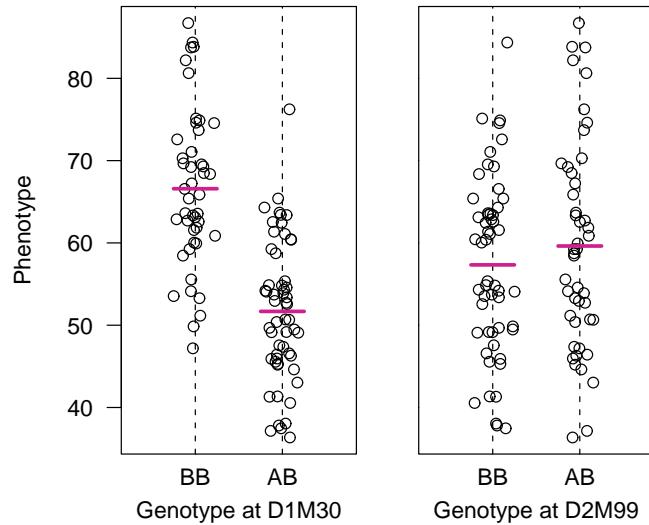
**Additivity:**  $\mu_{g_1, \dots, g_p} = \mu + \sum_{j=1}^p \Delta_j g_j$  ( $g_j = 1$  or  $0$ )

**Epistasis:** Any deviations from additivity.

# The simplest method: ANOVA

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- Split mice into groups according to genotype at a marker.
- Do a t-test / ANOVA.
- Repeat for each marker.
- Adjust for multiple testing



LOD score =  $\log_{10}$  likelihood ratio comparing single-QTL model to “no QTL anywhere.”

## ANOVA at marker loci

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### Advantages

- Simple.
- Easily incorporates covariates.
- Easily extended to more complex models.
- Doesn't require a genetic map.

### Disadvantages

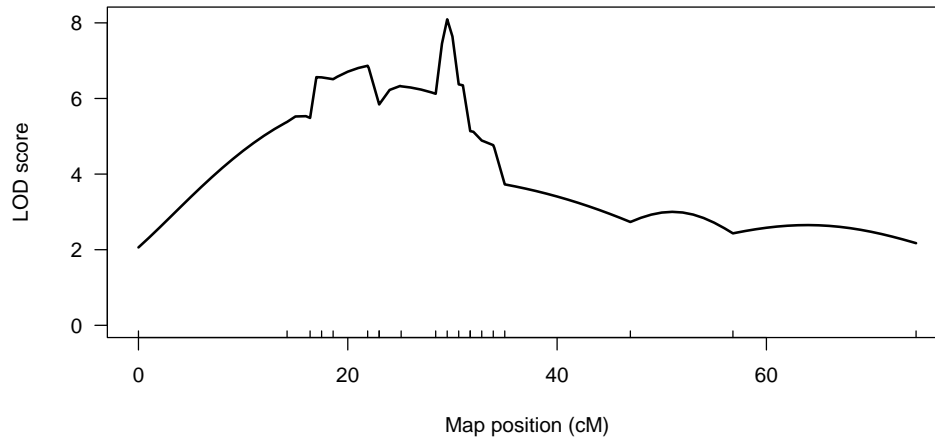
- Must exclude individuals with missing genotype data.
- Imperfect information about QTL location.
- Suffers in low density scans.
- **Only considers one QTL at a time.**



# Interval mapping (IM)

## Lander & Botstein (1989)

- Take account of missing genotype data
- Interpolate between markers
- Maximum likelihood under a mixture model



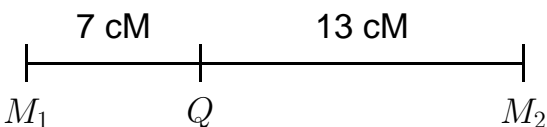
# Interval mapping (IM)

## Lander & Botstein (1989)

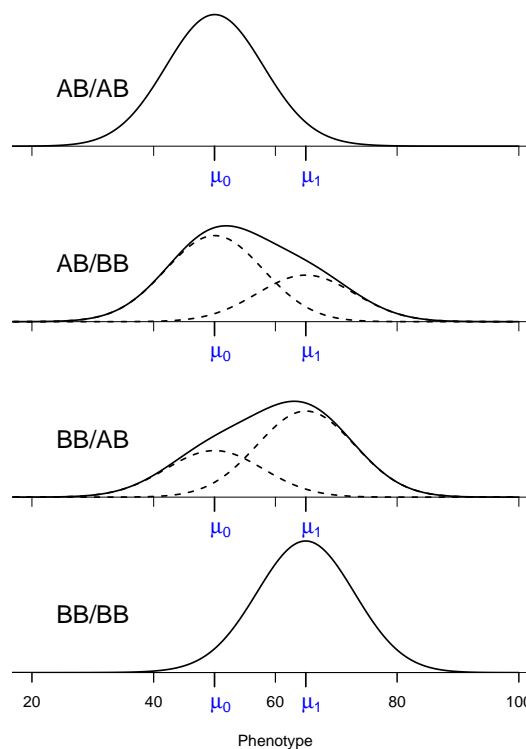
- Assume a **single** QTL model.
- Each position in the genome, one at a time, is posited as the putative QTL.
- Let  $z = 1/0$  if the (unobserved) QTL genotype is BB/AB.  
Assume  $y \sim N(\mu_z, \sigma)$
- Given genotypes at linked markers,  $y \sim$  mixture of normal dist'ns with mixing proportion  $\Pr(z = 1 | \text{marker data})$ :

		QTL genotype	
		BB	AB
$M_1$	$M_2$		
BB	BB	$(1 - r_L)(1 - r_R)/(1 - r)$	$r_L r_R/(1 - r)$
BB	AB	$(1 - r_L)r_R/r$	$r_L(1 - r_R)/r$
AB	BB	$r_L(1 - r_R)/r$	$(1 - r_L)r_R/r$
AB	AB	$r_L r_R/(1 - r)$	$(1 - r_L)(1 - r_R)/(1 - r)$

# The normal mixtures



- Two markers separated by 20 cM, with the QTL closer to the left marker.
- The figure at right show the distributions of the phenotype conditional on the genotypes at the two markers.
- The dashed curves correspond to the components of the mixtures.



## Interval mapping (continued)

Let  $p_i = \Pr(z_i = 1 | \text{marker data})$

$$y_i | z_i \sim N(\mu_{z_i}, \sigma^2)$$

$$\Pr(y_i | \text{marker data}, \mu_0, \mu_1, \sigma) = p_i f(y_i; \mu_1, \sigma) + (1 - p_i) f(y_i; \mu_0, \sigma)$$

$$\text{where } f(y; \mu, \sigma) = \exp[-(y - \mu)^2 / (2\sigma^2)] / \sqrt{2\pi\sigma^2}$$

**Log likelihood:**  $l(\mu_0, \mu_1, \sigma) = \sum_i \log \Pr(y_i | \text{marker data}, \mu_0, \mu_1, \sigma)$

Maximum likelihood estimates (**MLEs**) of  $\mu_0, \mu_1, \sigma$ :

values for which  $l(\mu_0, \mu_1, \sigma)$  is maximized.

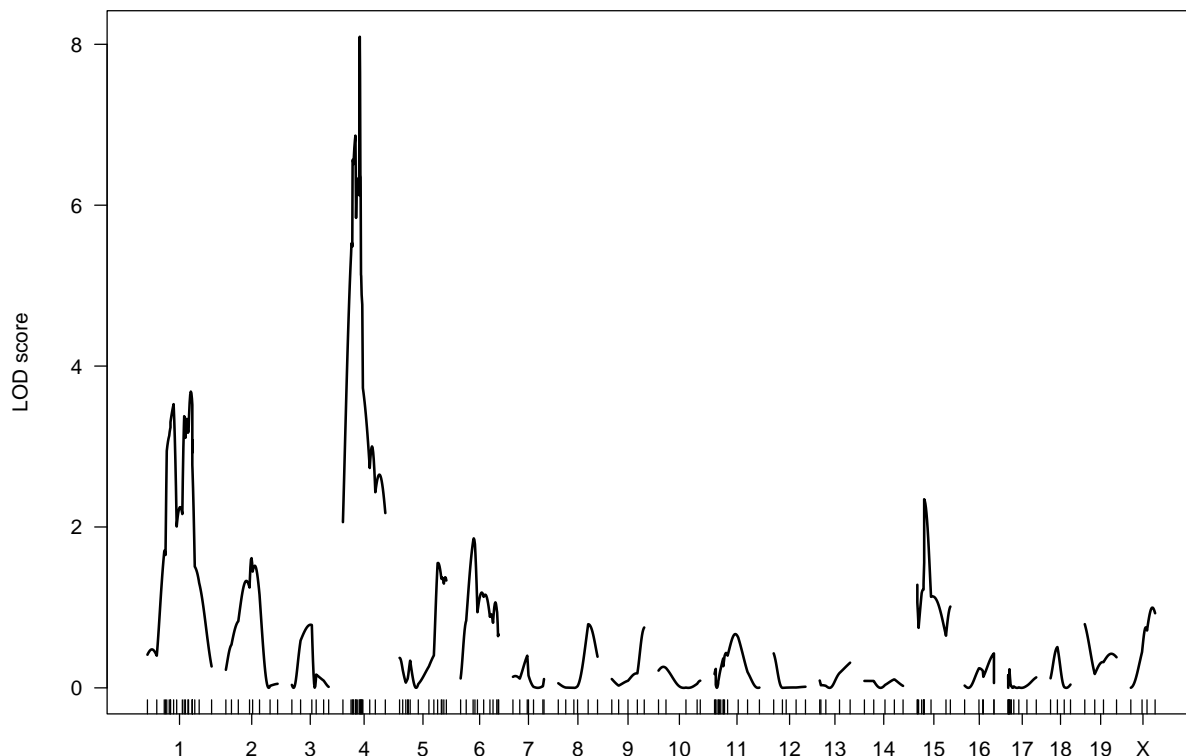
# LOD scores

The LOD score is a measure of the **strength of evidence** for the presence of a QTL at a particular location.

$$\begin{aligned}\text{LOD}(\gamma) &= \log_{10} \text{likelihood ratio comparing the hypothesis of a} \\ &\quad \text{QTL at position } \gamma \text{ versus that of no QTL} \\ &= \log_{10} \left\{ \frac{\Pr(y|\text{QTL at } \gamma, \hat{\mu}_{0\gamma}, \hat{\mu}_{1\gamma}, \hat{\sigma}_{\gamma})}{\Pr(y|\text{no QTL}, \hat{\mu}, \hat{\sigma})} \right\}\end{aligned}$$

$\hat{\mu}_{0\gamma}, \hat{\mu}_{1\gamma}, \hat{\sigma}_{\gamma}$  are the MLEs, assuming a single QTL at position  $\gamma$ .

No QTL model: The phenotypes are independent and identically distributed (iid)  $N(\mu, \sigma^2)$ .



# Interval mapping

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## Advantages

- Takes proper account of missing data.
- Allows examination of positions between markers.
- Gives improved estimates of QTL effects.
- Provides pretty graphs.

## Disadvantages

- Increased computation time.
- Requires specialized software.
- Difficult to generalize.
- **Only considers one QTL at a time.**

## Haley-Knott regression

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A quick approximation to Interval Mapping.

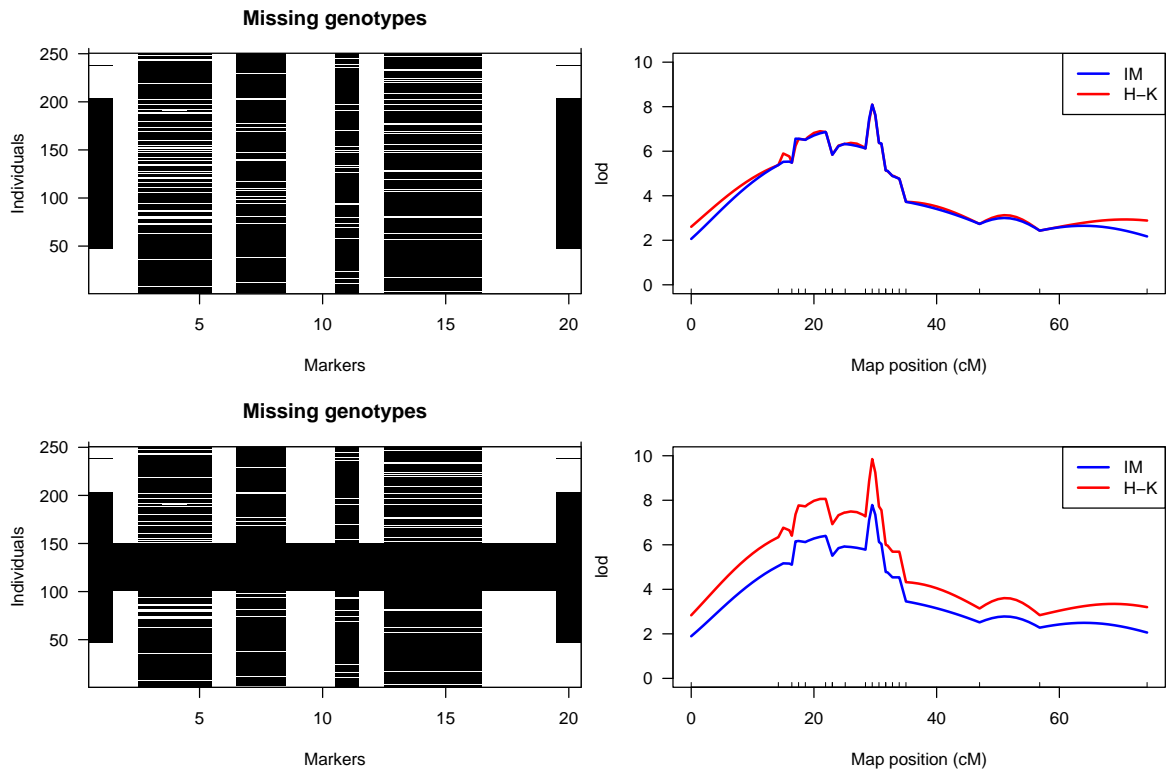
$$E(y \mid \text{QTL} = q) = \mu + \beta \mathbf{1}\{q = \text{AB}\}$$

$$E(y \mid \text{marker data}) = \mu + \beta \Pr(\text{QTL} = \text{AB} \mid \text{marker data})$$

- Regress  $y$  on  $\Pr(\text{QTL} = \text{AB} \mid \text{marker data})$ .
- **Pretend** that the residual variation is normally distributed.
- Calculate

$$\text{LOD}(\gamma) = (n/2) \log_{10} \left\{ \frac{\text{RSS}_0}{\text{RSS}_a(\gamma)} \right\}$$

# Example



## LOD thresholds

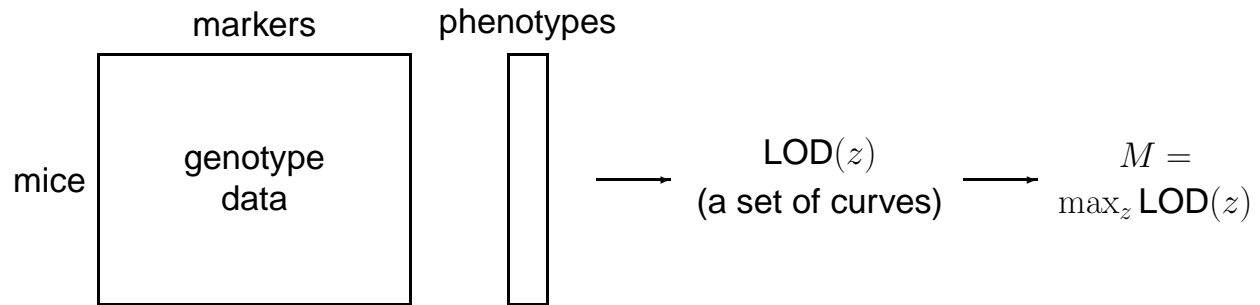
Large LOD scores indicate evidence for the presence of a QTL

**Question:** How large is large?

**LOD threshold** = 95 %ile of distr'n of max LOD, genome-wide, if there are no QTLs anywhere

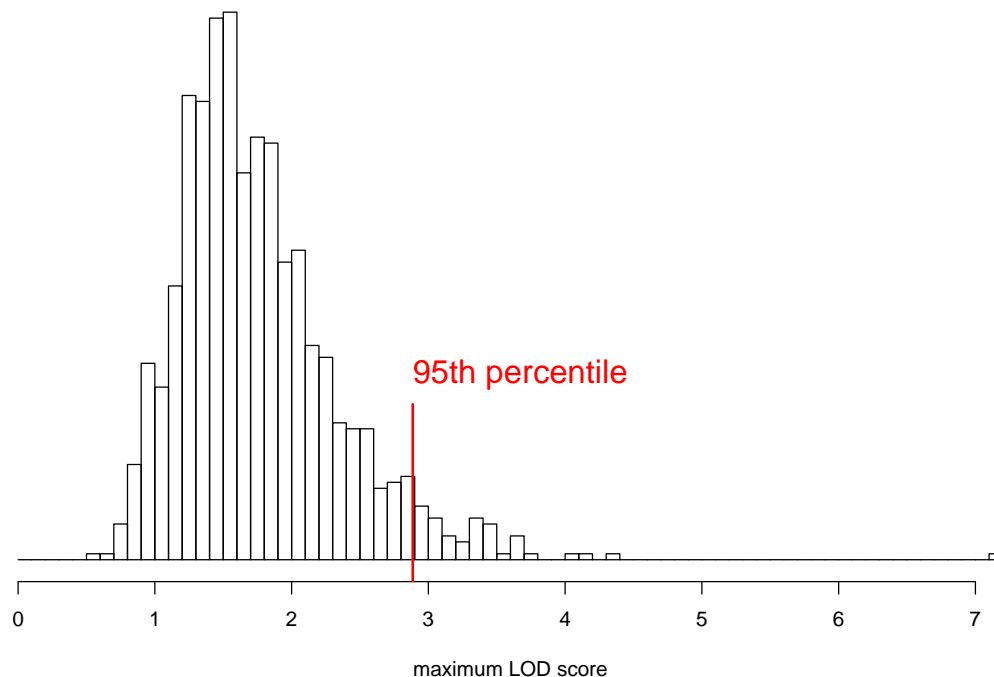
- Derivation:**
- Analytical calculations (L & B 1989)
  - Simulations (L & B 1989)
  - Permutation tests (Churchill & Doerge 1994)

# Permutation tests

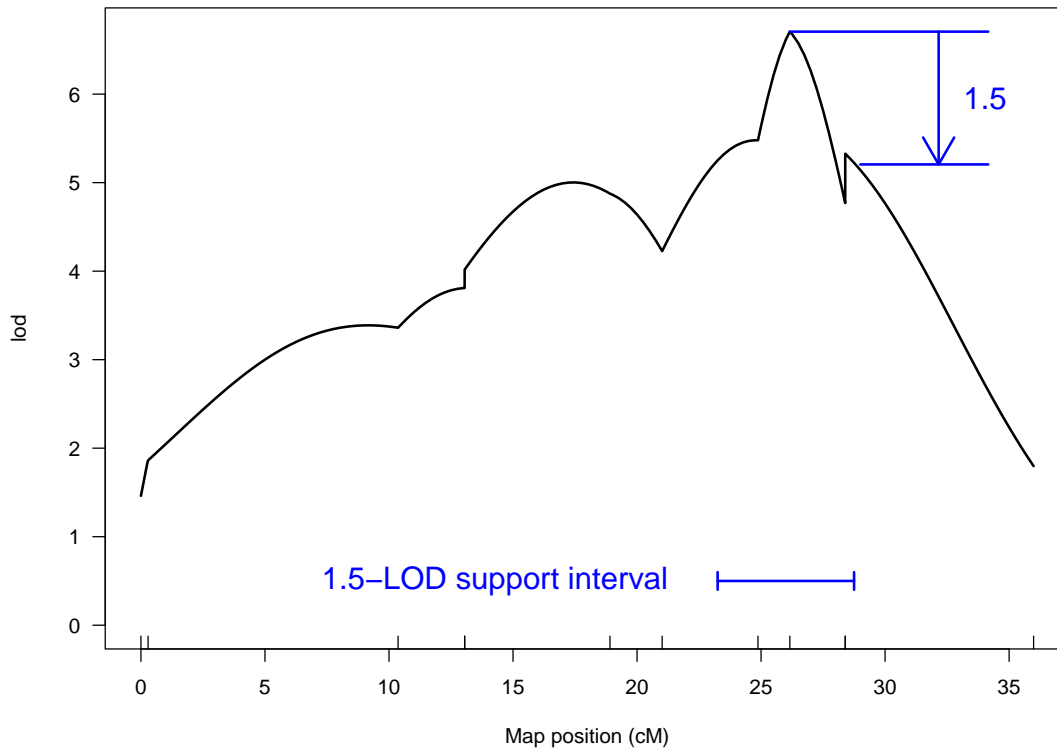


- Permute/shuffle the phenotypes; keep the genotype data intact.
- Calculate  $\text{LOD}^*(z) \rightarrow M^* = \max_z \text{LOD}^*(z)$
- We wish to compare the observed  $M$  to the distribution of  $M^*$ .
- $\Pr(M^* \geq M)$  is a genome-wide P-value.
- The 95th %ile of  $M^*$  is a genome-wide LOD threshold.
- We can't look at all  $n!$  possible permutations, but a random set of 1000 is feasible and provides reasonable estimates of P-values and thresholds.
- **Value:** conditions on observed phenotypes, marker density, and pattern of missing data; doesn't rely on normality assumptions or asymptotics.

## Permutation distribution

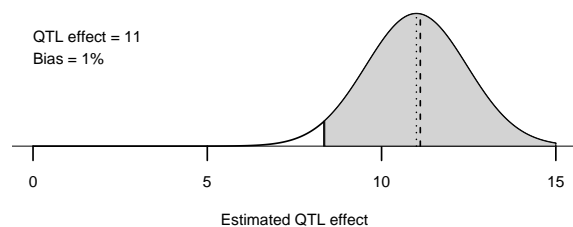
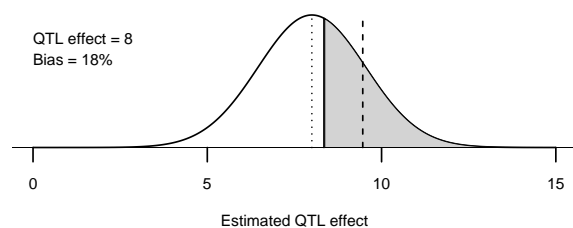
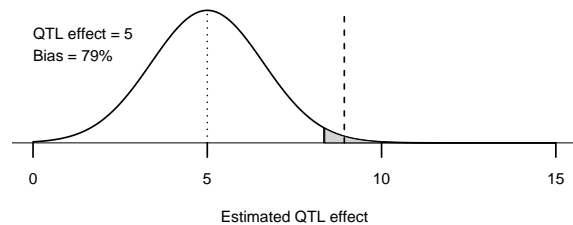


# 1.5-LOD support interval



## Selection bias

- The estimated effect of a QTL will vary somewhat from its true effect.
- Only when the estimated effect is large will the QTL be detected.
- Among those experiments in which the QTL is detected, the estimated QTL effect will be, on average, larger than its true effect.
- This is **selection bias**.
- Selection bias is largest in QTLs with small or moderate effects.
- The true effects of QTLs that we identify are likely smaller than was observed.



# Implications of selection bias

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- Estimated % variance explained by identified QTLs
- Repeating an experiment
- Congenics
- Marker-assisted selection

## Multiple QTL methods

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### Why consider multiple QTLs at once?

- Reduce residual variation.
- Separate linked QTLs.
- Investigate interactions between QTLs (epistasis).

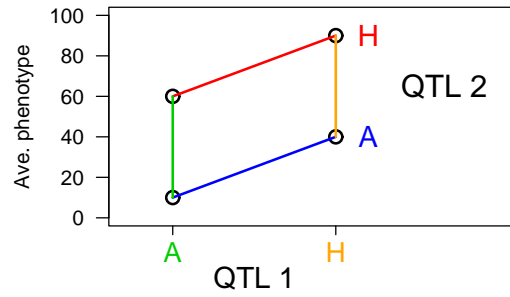
### Issues:

- Missing genotype information
- The model selection problem

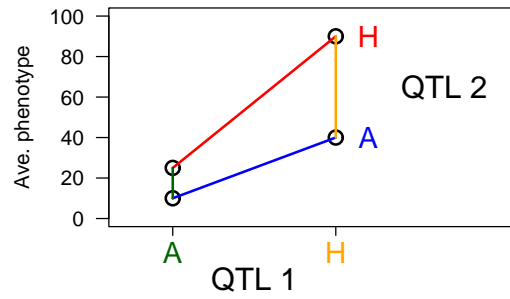


# Epistasis in a backcross

Additive QTLs

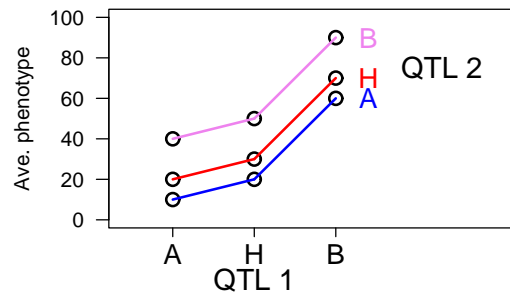


Interacting QTLs

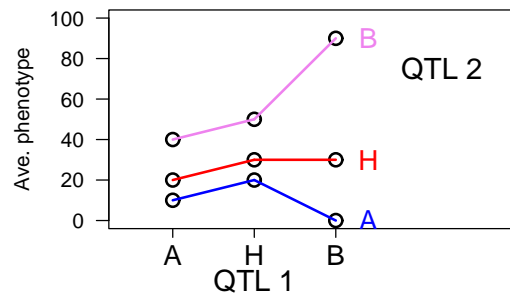


# Epistasis in an intercross

Additive QTLs



Interacting QTLs



# Two-dimensional genome scan

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Consider each pair of positions,  $(\gamma_1, \gamma_2)$

## Models

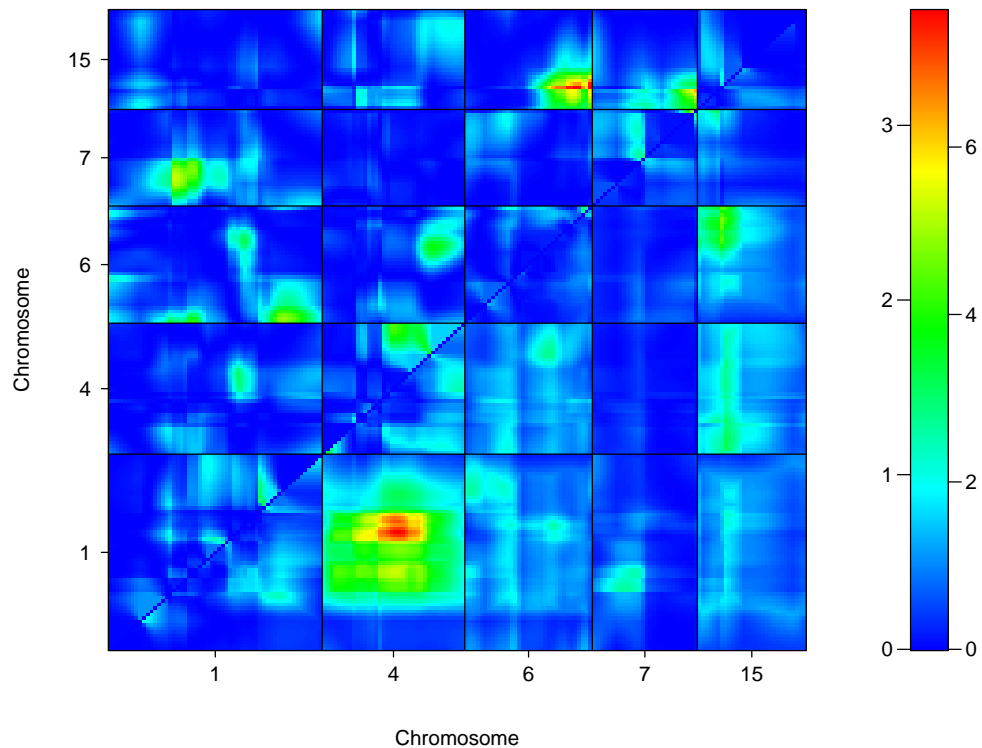
- Full
- Additive
- QTL 1
- QTL 2
- Null

## Possible comparisons

- Full vs. null
- Full vs. additive
- Full vs. Best of QTL 1 & 2
- Add've vs. Best of QTL 1 & 2

## Example

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## The X chromosome

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In a backcross, the X chromosome may or may not be segregating.

$$(A \times B) \times A$$

Females:  $X_{A \cdot B} X_A$

Males:  $X_{A \cdot B} Y_A$

$$A \times (A \times B)$$

Females:  $X_A X_A$

Males:  $X_A Y_B$

## The X chromosome

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In an intercross, one must pay attention to the **paternal grandmother's genotype**.

$$(A \times B) \times (A \times B) \quad \text{or} \quad (B \times A) \times (A \times B)$$

Females:  $X_{A \cdot B} X_A$

Males:  $X_{A \cdot B} Y_B$

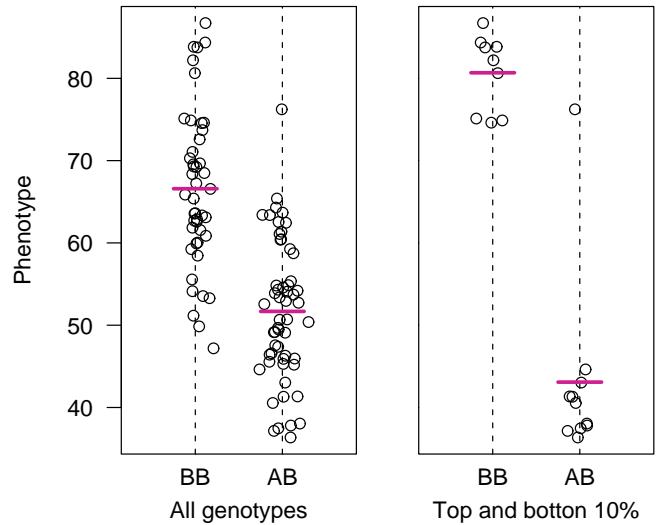
$$(A \times B) \times (B \times A) \quad \text{or} \quad (B \times A) \times (B \times A)$$

Females:  $X_{A \cdot B} X_B$

Males:  $X_{A \cdot B} Y_A$

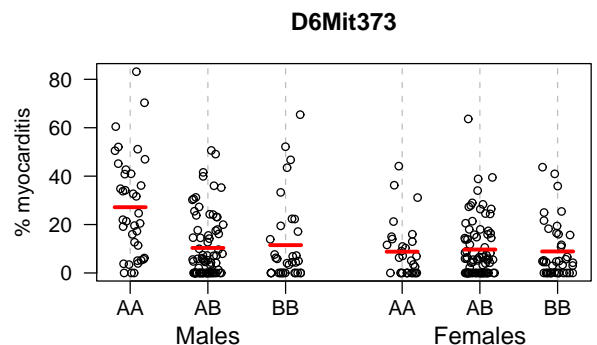
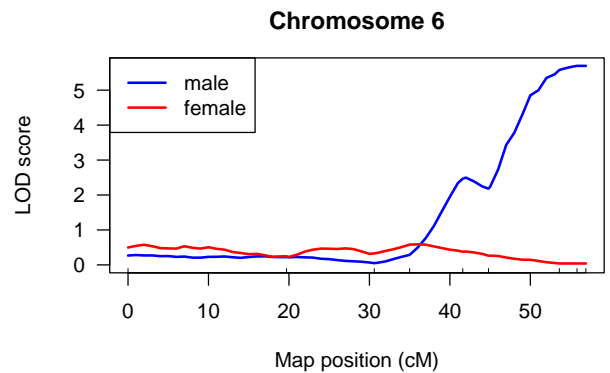
# Selective genotyping

- Save effort by only typing the most informative individuals (say, top & bottom 10%).
- Useful in context of a **single, inexpensive** trait.
- Tricky to estimate the effects of QTLs: use IM with **all** phenotypes.
- Can't get at interactions.
- Likely better to also genotype some random portion of the rest of the individuals.



# Covariates

- **Examples:** treatment, sex, litter, lab, age.
- Control residual variation.
- Avoid confounding.
- Look for QTL  $\times$  environ't interactions
- Adjust before interval mapping (IM) versus adjust within IM.



## Non-normal traits

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- Standard interval mapping assumes normally distributed residual variation. (Thus the phenotype distribution is a mixture of normals.)
- **In reality:** we see dichotomous traits, counts, skewed distributions, outliers, and all sorts of odd things.
- Interval mapping, with LOD thresholds derived from permutation tests, generally performs just fine anyway.
- Alternatives to consider:
  - Nonparametric approaches (Kruglyak & Lander 1995)
  - Transformations (e.g., log, square root)
  - Specially-tailored models (e.g., a generalized linear model, the Cox proportional hazard model, and the model in Broman (2003))

## Check data integrity

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The success of QTL mapping depends crucially on the integrity of the data.

- Segregation distortion
- Genetic maps / marker positions
- Genotyping errors (tight double crossovers)
- Phenotype distribution / outliers
- Residual analysis

# Summary I

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- **ANOVA** at marker loci (aka marker regression) is simple and easily extended to include covariates or accommodate complex models.
- **Interval mapping** improves on ANOVA by allowing inference of QTLs to positions between markers and taking proper account of missing genotype data.
- ANOVA and IM consider only single-QTL models. **Multiple QTL methods** allow the better separation of linked QTLs and are necessary for the investigation of epistasis.
- Statistical significance of LOD peaks requires consideration of the maximum LOD score, genome-wide, under the null hypothesis of no QTLs. **Permutation tests** are extremely useful for this.
- **1.5-LOD support intervals** indicate the plausible location of a QTL.
- Estimates of QTL effects are subject to **selection bias**. Such estimated effects are often too large.

# Summary II

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- The **X chromosome** must be dealt with specially, and can be tricky.
- **Study your data**. Look for errors in the genetic map, genotyping errors and phenotype outliers. But don't worry about them too much.
- **Selective genotyping** can save you time and money, but proceed with caution.
- **Study your data**. The consideration of covariates may reveal extremely interesting phenomena.
- Interval mapping works reasonably well even with **non-normal traits**. But consider transformations or specially-tailored models. If interval mapping software is not available for your preferred model, start with some version of ANOVA.

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