

R/qtl workshop

(part 1)

Karl Broman

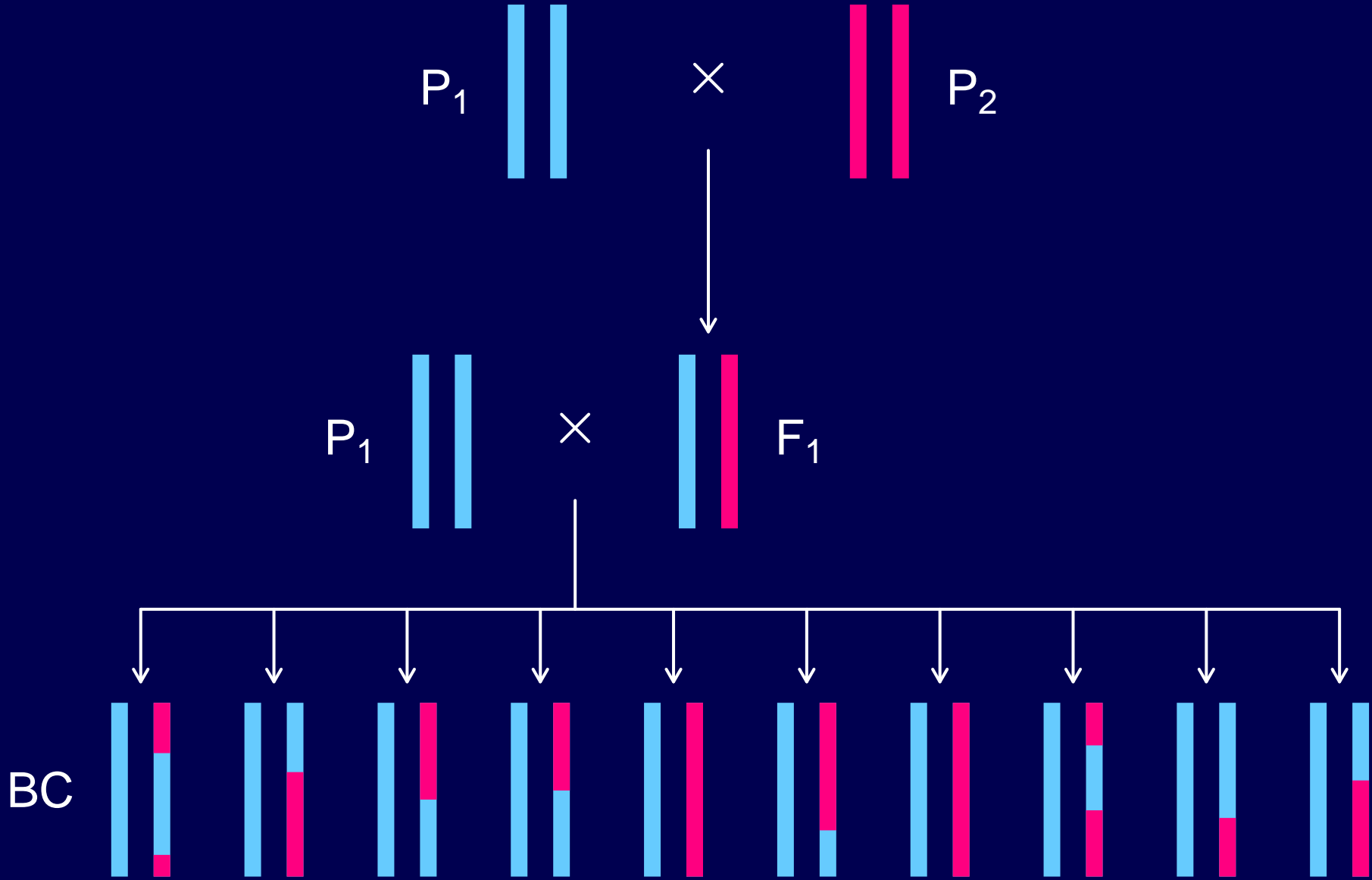
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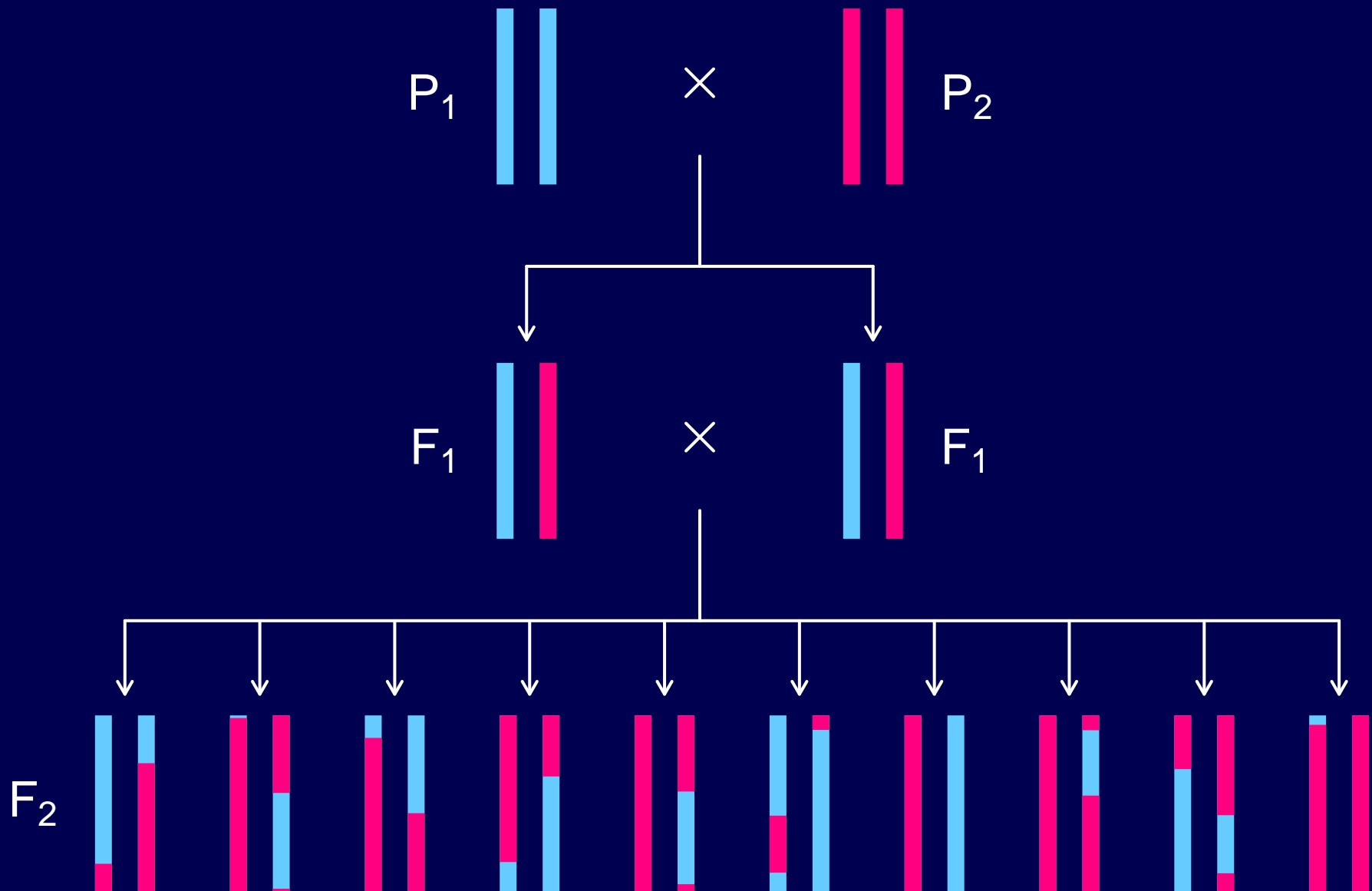
github.com/kbroman

@kwbroman

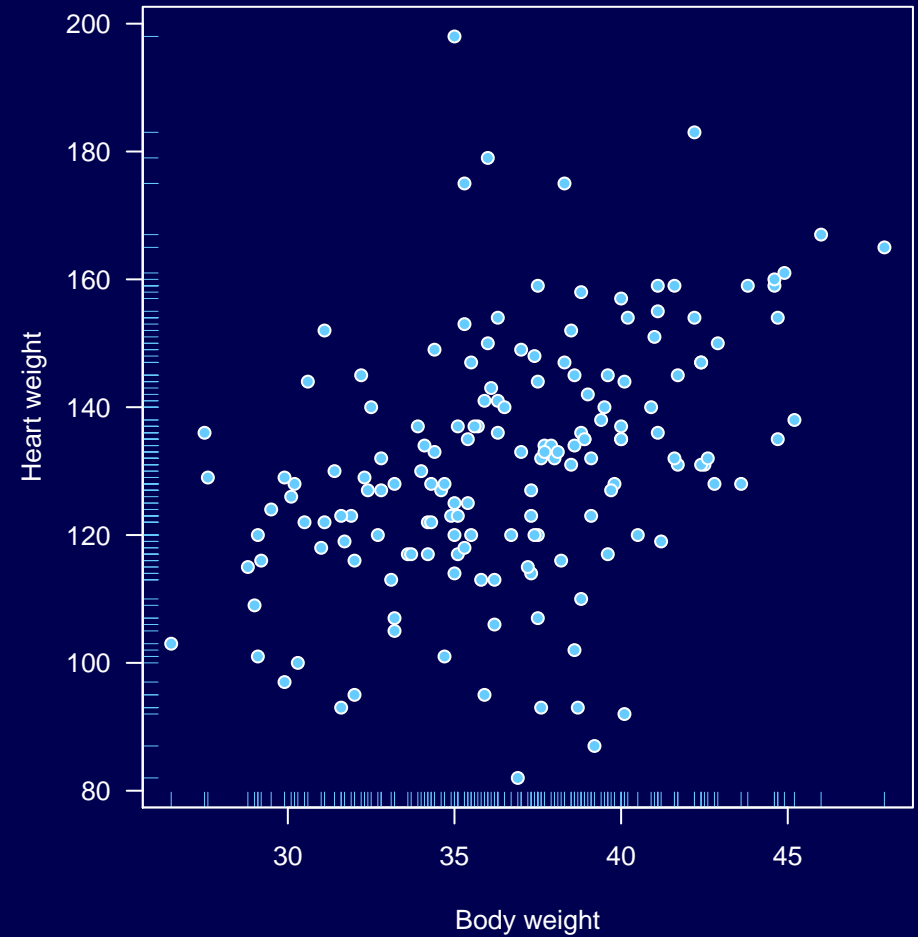
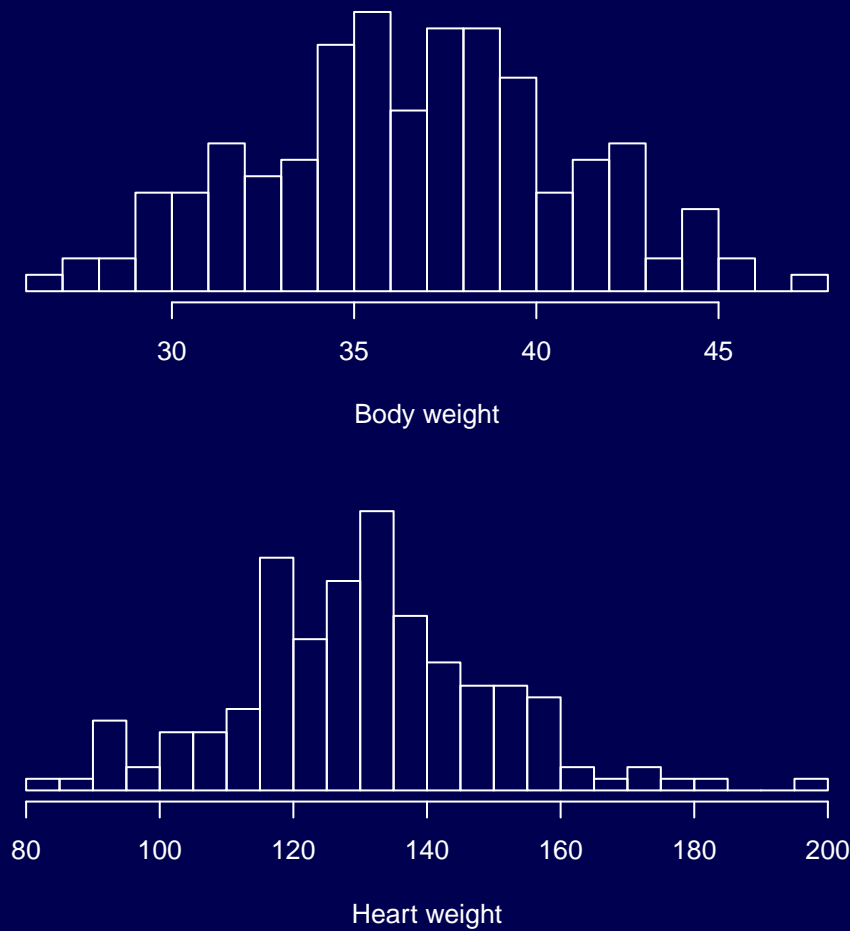
Backcross



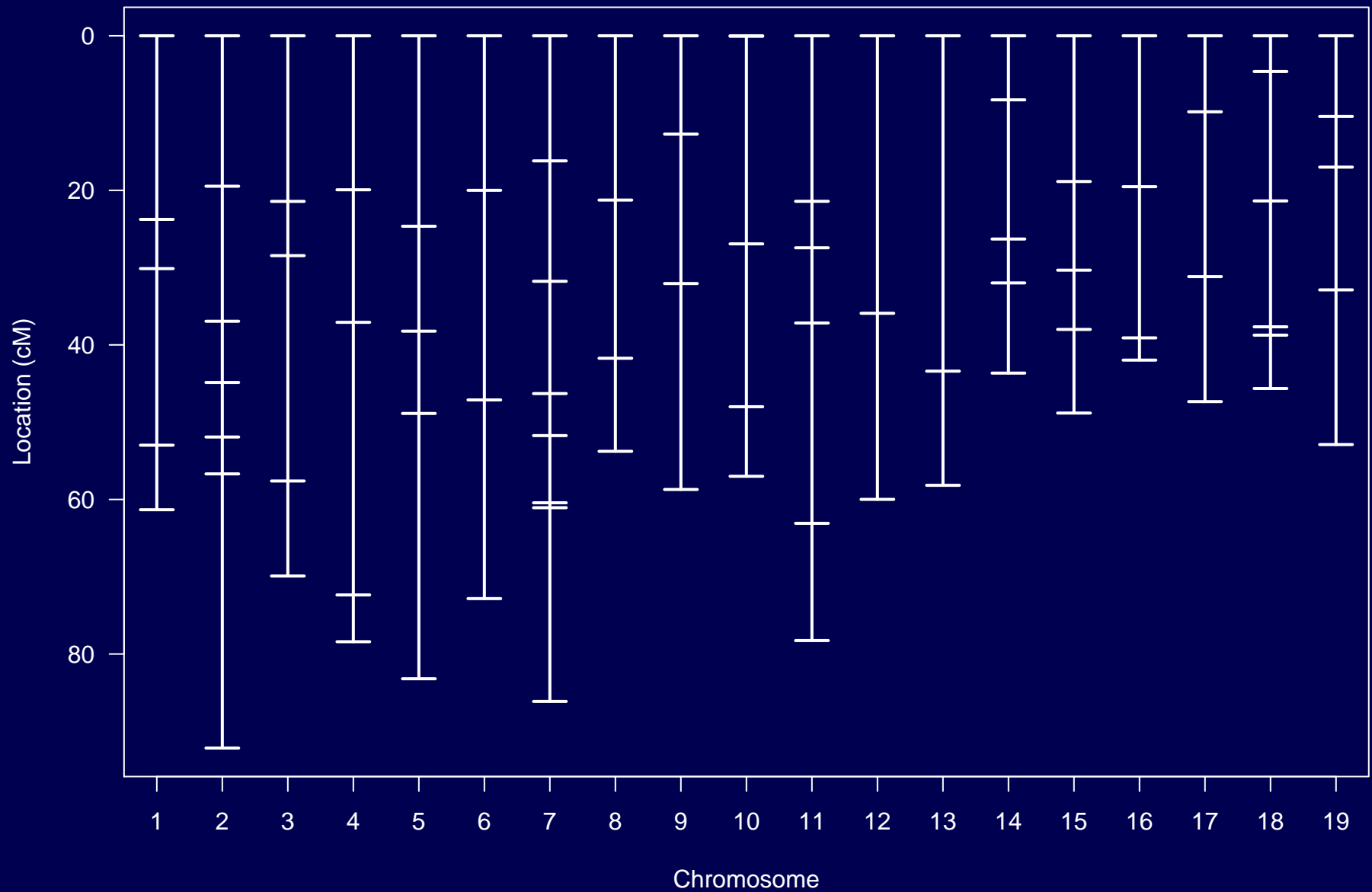
Intercross



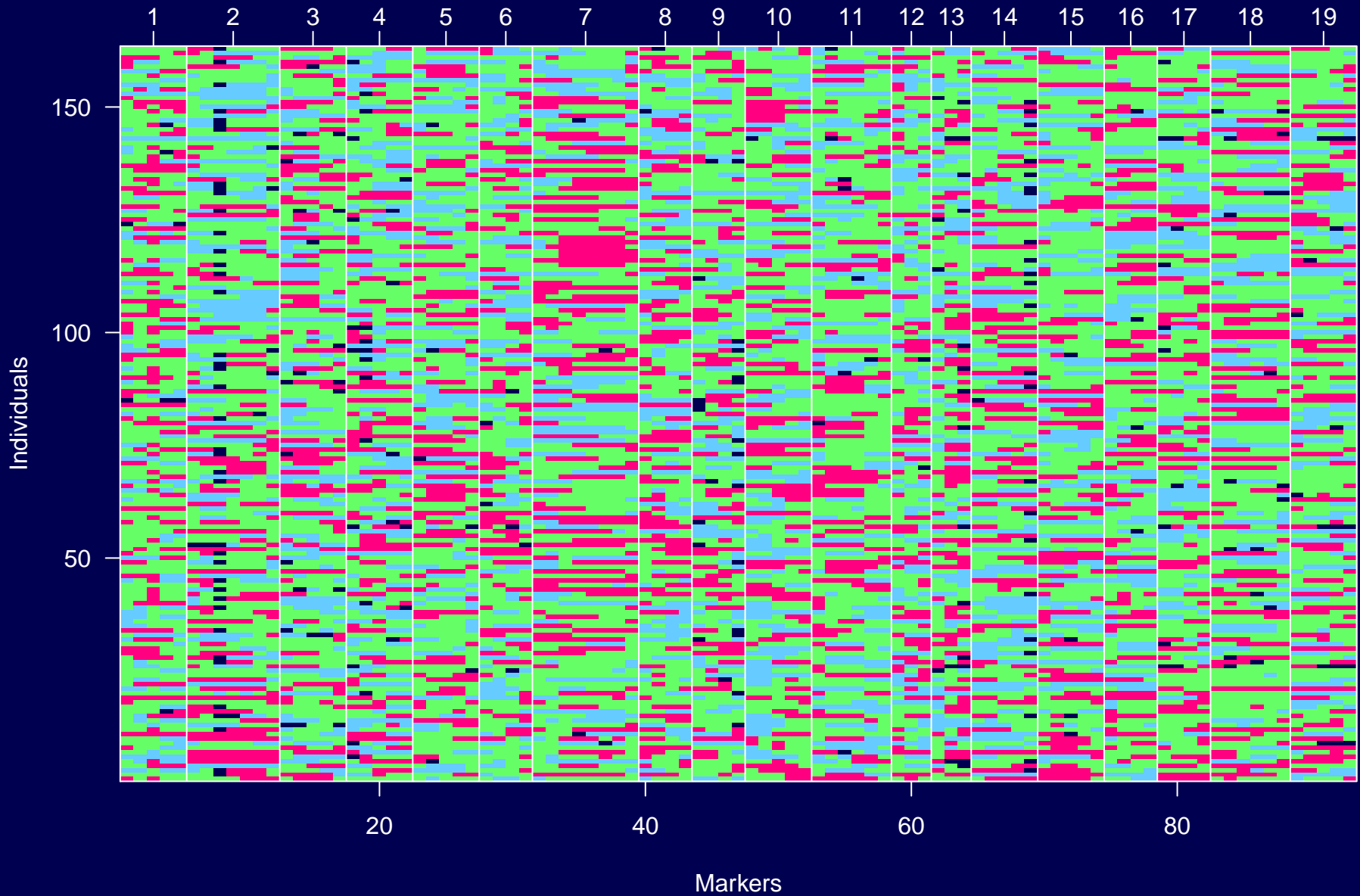
Phenotype data



Genetic map



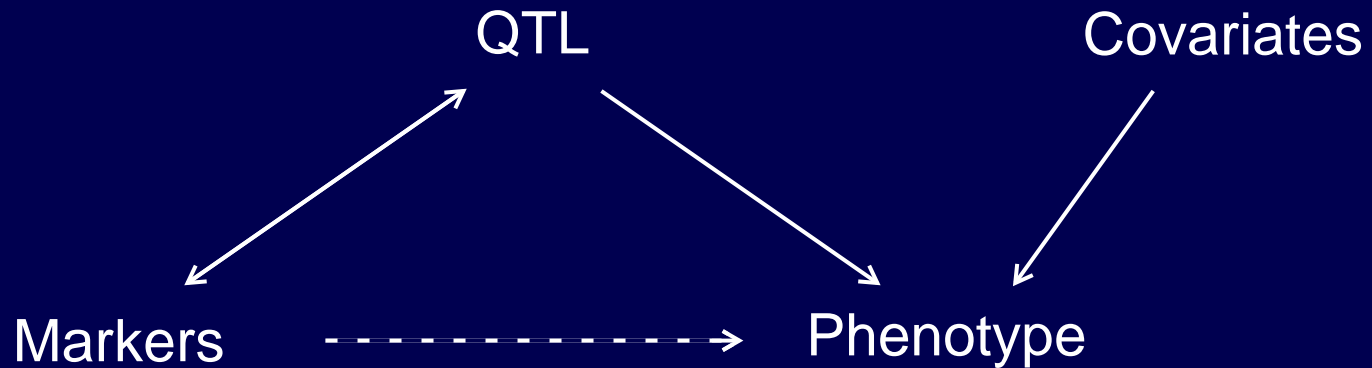
Genotype data



Goals

- Identify quantitative trait loci (QTL)
(and interactions among QTL)
- Interval estimates of QTL location
- Estimated QTL effects

Statistical structure



The missing data problem:

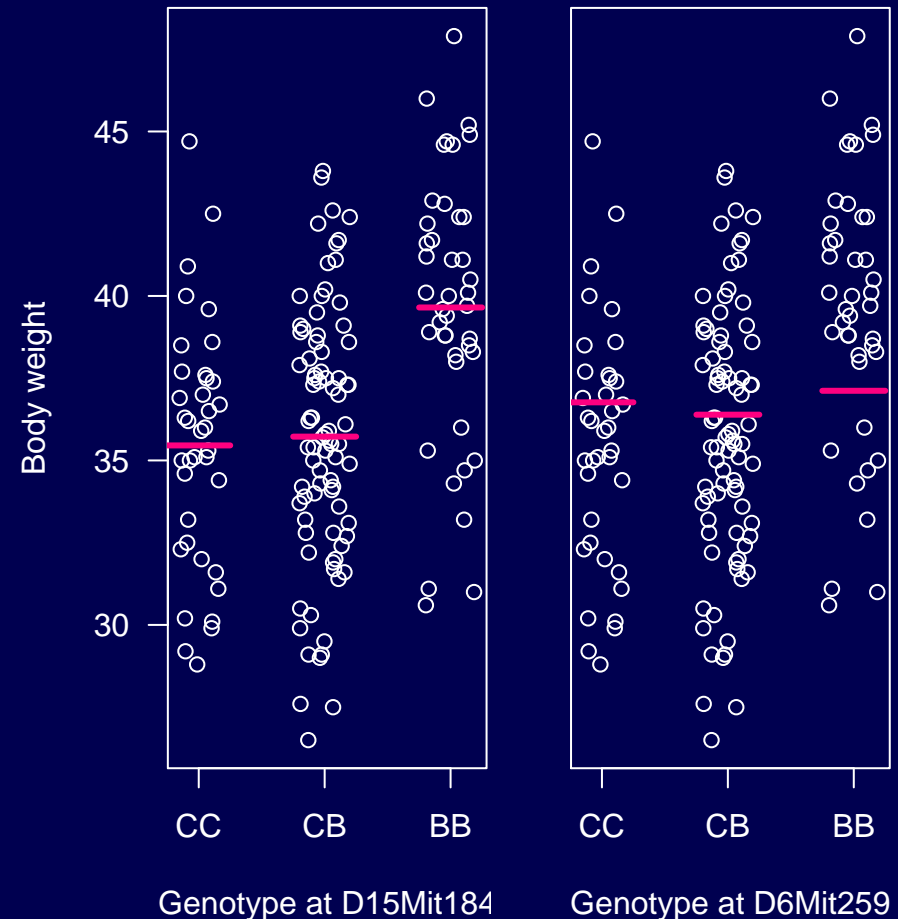
Markers \longleftrightarrow QTL

The model selection problem:

QTL, covariates \longrightarrow phenotype

ANOVA at marker loci

- Also known as **marker regression**.
- Split mice into groups according to genotype at a marker.
- Do a t-test / ANOVA.
- Repeat for each marker.



[R/ql]

ANOVA at marker loci

Advantages

- Simple.
- Easily incorporates covariates.
- Easily extended to more complex models.
- Doesn't require a genetic map.

Disadvantages

- Must exclude individuals with missing genotype data.
- Imperfect information about QTL location.
- Suffers in low density scans.
- Only considers one QTL at a time.

Interval mapping

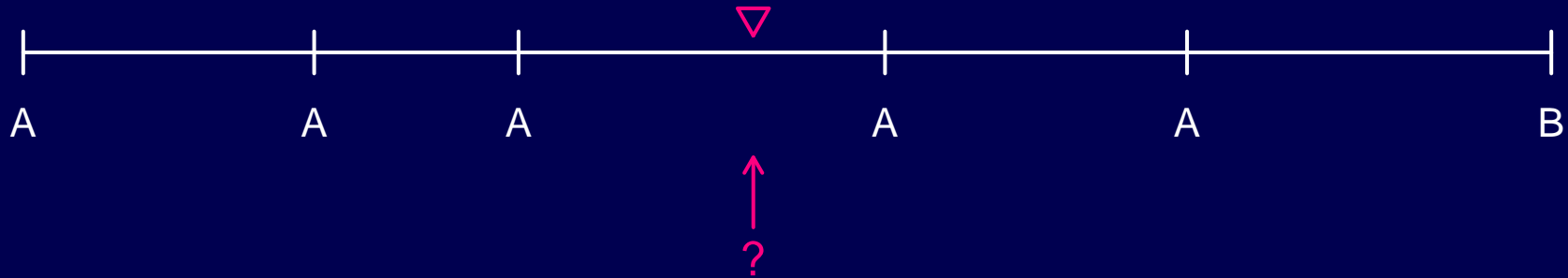
Lander & Botstein (1989)

- Assume a **single** QTL model.
- Each position in the genome, one at a time, is posited as the putative QTL.
- Let $q = 1/0$ if the (unobserved) QTL genotype is BB/AB.
(Or 2/1/0 if the QTL genotype is BB/AB/AA in an intercross.)

Assume $y|q \sim N(\mu_q, \sigma)$

- Given genotypes at linked markers, $y \sim$ mixture of normal dist'ns with mixing proportions $\Pr(q \mid \text{marker data})$

Genotype probabilities



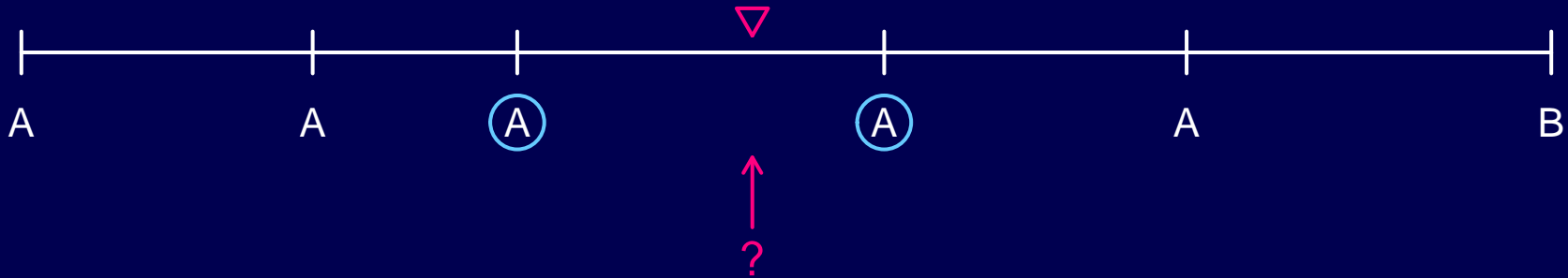
Calculate $\Pr(q \mid \text{marker data})$, assuming

- No crossover interference
- No genotyping errors

Or use the **hidden Markov model (HMM)** technology

- To allow for genotyping errors
- To incorporate dominant markers
- (Still assume no crossover interference.)

Genotype probabilities



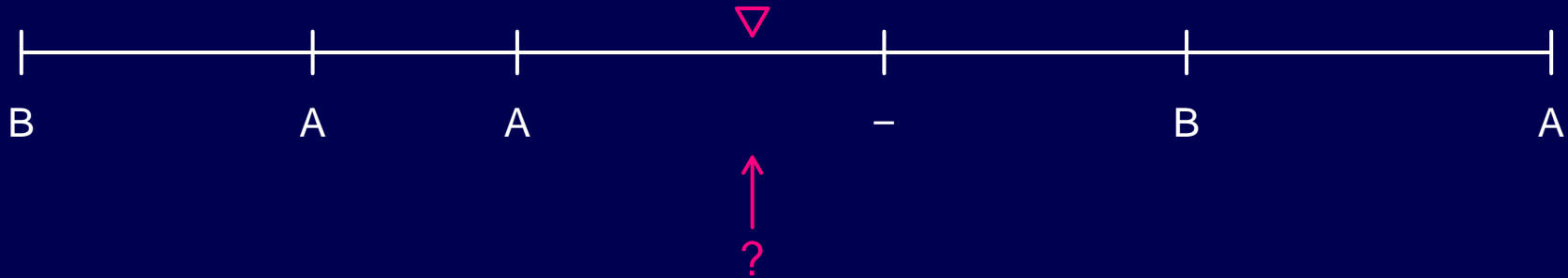
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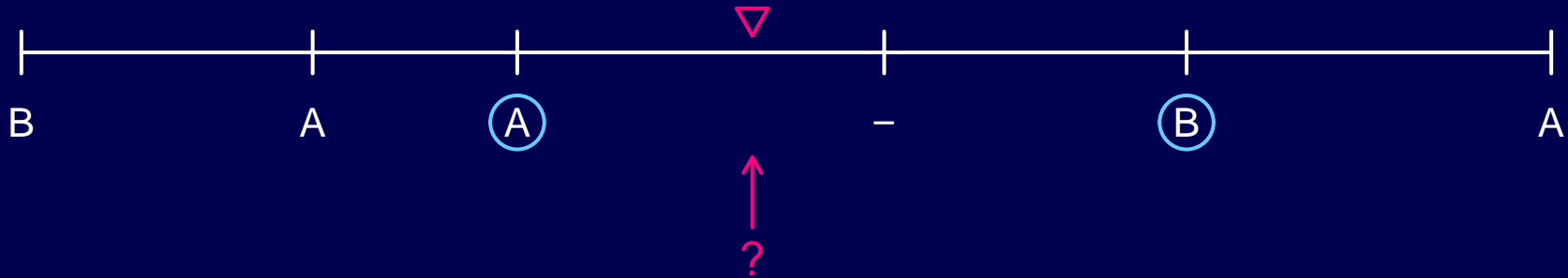
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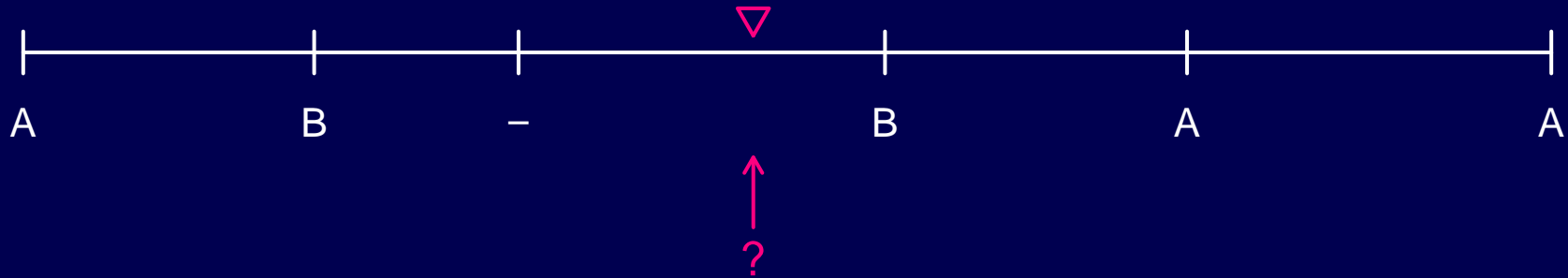
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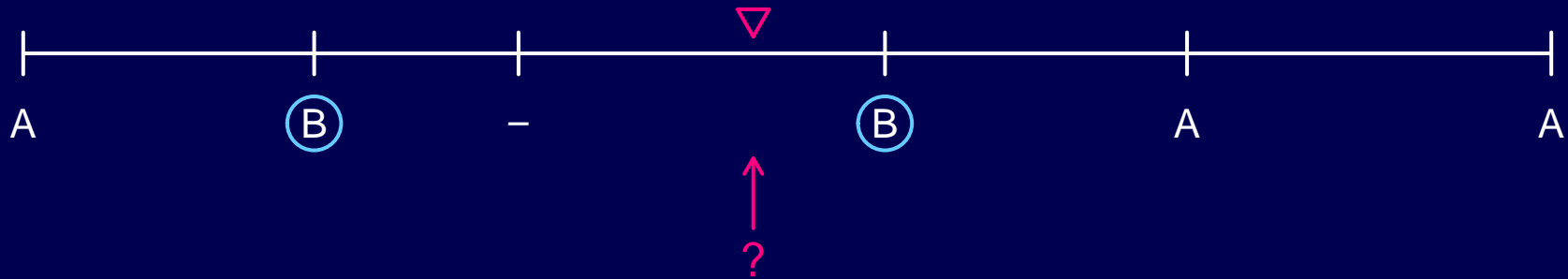
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Genotype probabilities



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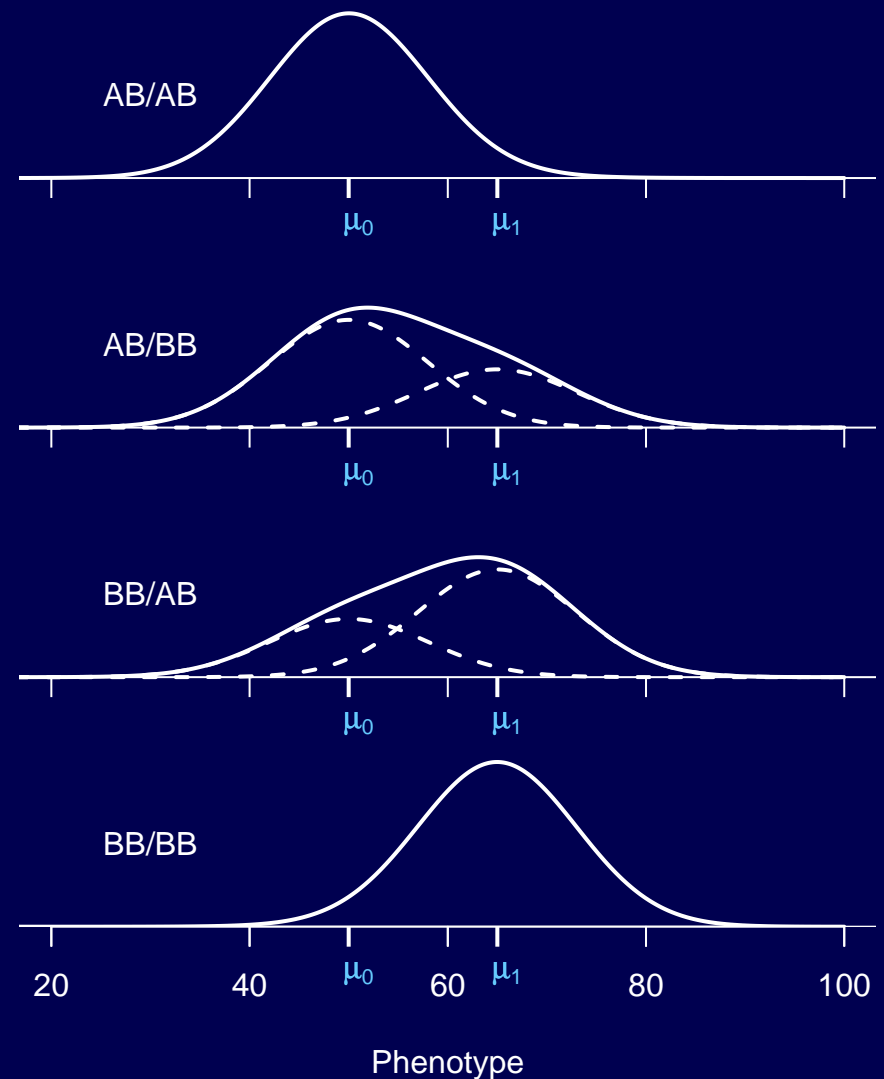
Or use the **hidden Markov model (HMM)** technology

- To allow for genotyping errors
- To incorporate dominant markers
- (Still assume no crossover interference.)

The normal mixtures



- Two markers separated by 20 cM, with the QTL closer to the left marker.
- The figure at right shows the distributions of the phenotype conditional on the genotypes at the two markers.
- The dashed curves correspond to the components of the mixtures.



Interval mapping

Let $p_{ij} = \Pr(q_i = j | \text{marker data})$

$$y_i | q_i \sim N(\mu_{q_i}, \sigma^2)$$

$$\Pr(y_i | \text{marker data}, \mu_0, \mu_1, \sigma) = \sum_j p_{ij} f(y_i; \mu_j, \sigma)$$

$$\text{where } f(y; \mu, \sigma) = \exp[-(y - \mu)^2 / (2\sigma^2)] / \sqrt{2\pi\sigma^2}$$

Log likelihood: $l(\mu_0, \mu_1, \sigma) = \sum_i \log \Pr(y_i | \text{marker data}, \mu_0, \mu_1, \sigma)$

Maximum likelihood estimates (**MLEs**) of μ_0, μ_1, σ :

values for which $l(\mu_0, \mu_1, \sigma)$ is maximized.

EM algorithm

Dempster et al. (1977)

E step:

$$\begin{aligned}\text{Let } w_{ij}^{(k)} &= \Pr(q_i = j | y_i, \text{marker data}, \hat{\mu}_0^{(k-1)}, \hat{\mu}_1^{(k-1)}, \hat{\sigma}^{(k-1)}) \\ &= \frac{p_{ij} f(y_i; \hat{\mu}_j^{(k-1)}, \hat{\sigma}^{(k-1)})}{\sum_j p_{ij} f(y_i; \hat{\mu}_j^{(k-1)}, \hat{\sigma}^{(k-1)})}\end{aligned}$$

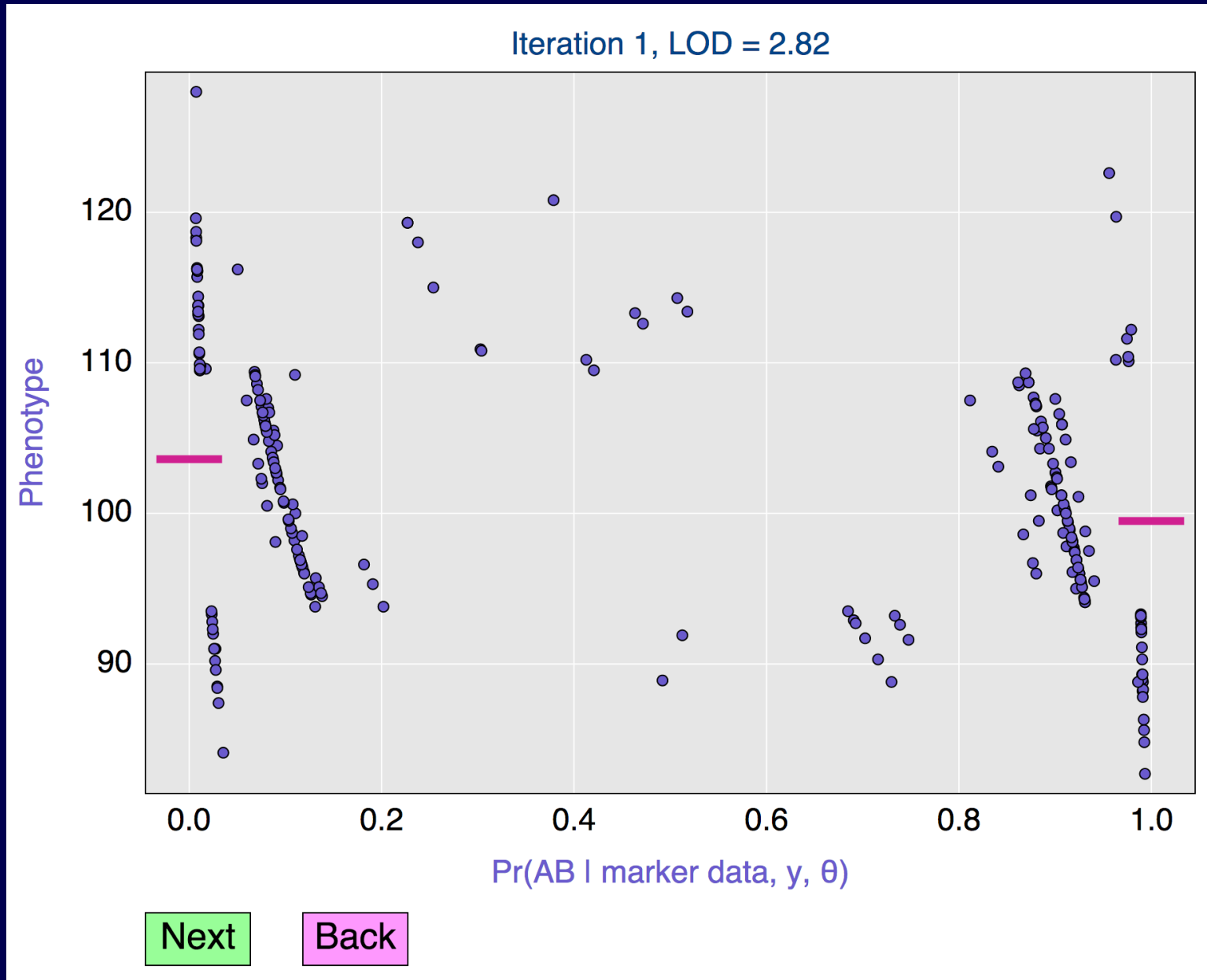
M step:

$$\begin{aligned}\text{Let } \hat{\mu}_j^{(k)} &= \sum_i y_i w_{ij}^{(k)} / \sum_i w_{ij}^{(k)} \\ \hat{\sigma}^{(k)} &= \sqrt{\sum_i \sum_j w_{ij}^{(k)} (y_i - \hat{\mu}_j^{(k)})^2 / n}\end{aligned}$$

The algorithm:

Start with $w_{ij}^{(1)} = p_{ij}$; iterate the E & M steps until convergence.

Interactive illustration



LOD scores

The LOD score is a measure of the **strength of evidence** for the presence of a QTL at a particular location.

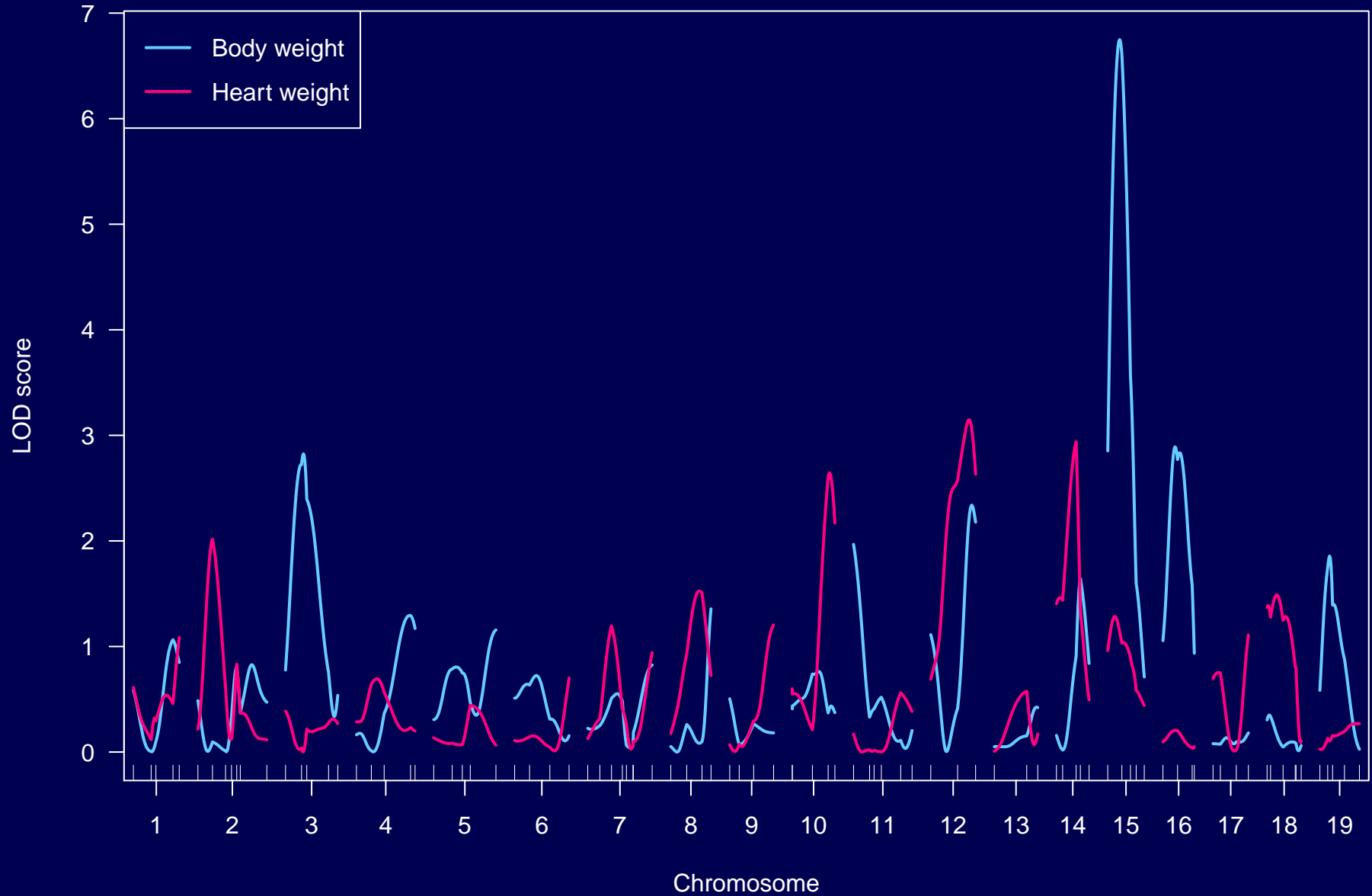
$\text{LOD}(\lambda) = \log_{10}$ likelihood ratio comparing the hypothesis of a QTL at position λ versus that of no QTL

$$= \log_{10} \left\{ \frac{\text{Pr}(\mathbf{y} | \text{QTL at } \lambda, \hat{\mu}_{0\lambda}, \hat{\mu}_{1\lambda}, \hat{\sigma}_{\lambda})}{\text{Pr}(\mathbf{y} | \text{no QTL}, \hat{\mu}, \hat{\sigma})} \right\}$$

$\hat{\mu}_{0\lambda}, \hat{\mu}_{1\lambda}, \hat{\sigma}_{\lambda}$ are the MLEs, assuming a single QTL at position λ .

No QTL model: The phenotypes are independent and identically distributed (iid) $N(\mu, \sigma^2)$.

LOD curves



[R/qtI]

Interval mapping

Advantages

- Takes proper account of missing data.
- Allows examination of positions between markers.
- Gives improved estimates of QTL effects.
- Provides pretty graphs.

Disadvantages

- Increased computation time.
- Requires specialized software.
- Difficult to generalize.
- Only considers one QTL at a time.

Haley-Knott regression

A quick approximation to Interval Mapping.

$$E(y_i|q_i) = \mu_q$$

$$\begin{aligned} E(y_i|M_i) &= E[E(y_i|q_i) |M_i] = \sum_j \Pr(q = j|M_i)\mu_j \\ &= \sum_j p_{ij}\mu_j \end{aligned}$$

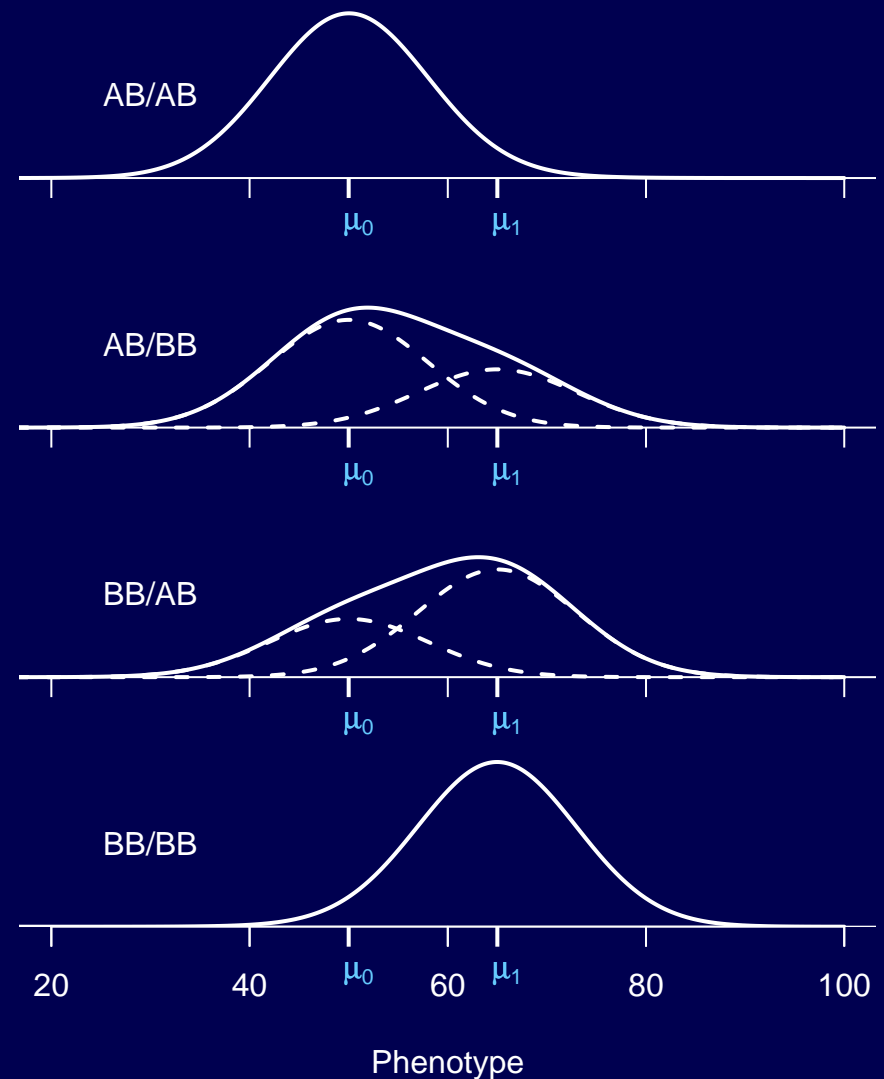
Regress y on p_i , pretending the residual variation is normally distributed (with constant variance).

$$\text{LOD} = \frac{n}{2} \log_{10} \left(\frac{\text{RSS}_0}{\text{RSS}_1} \right)$$

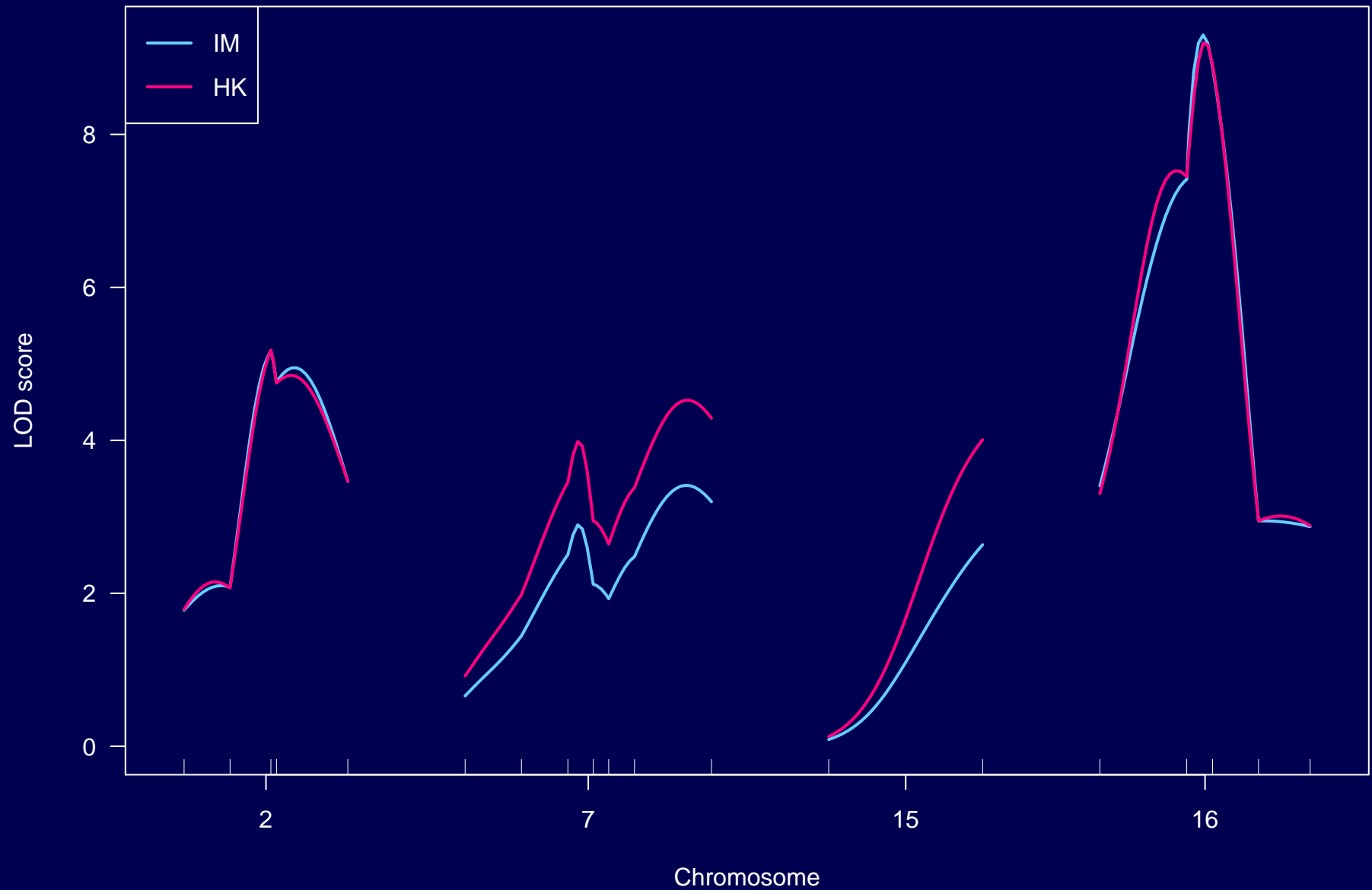
The normal mixtures



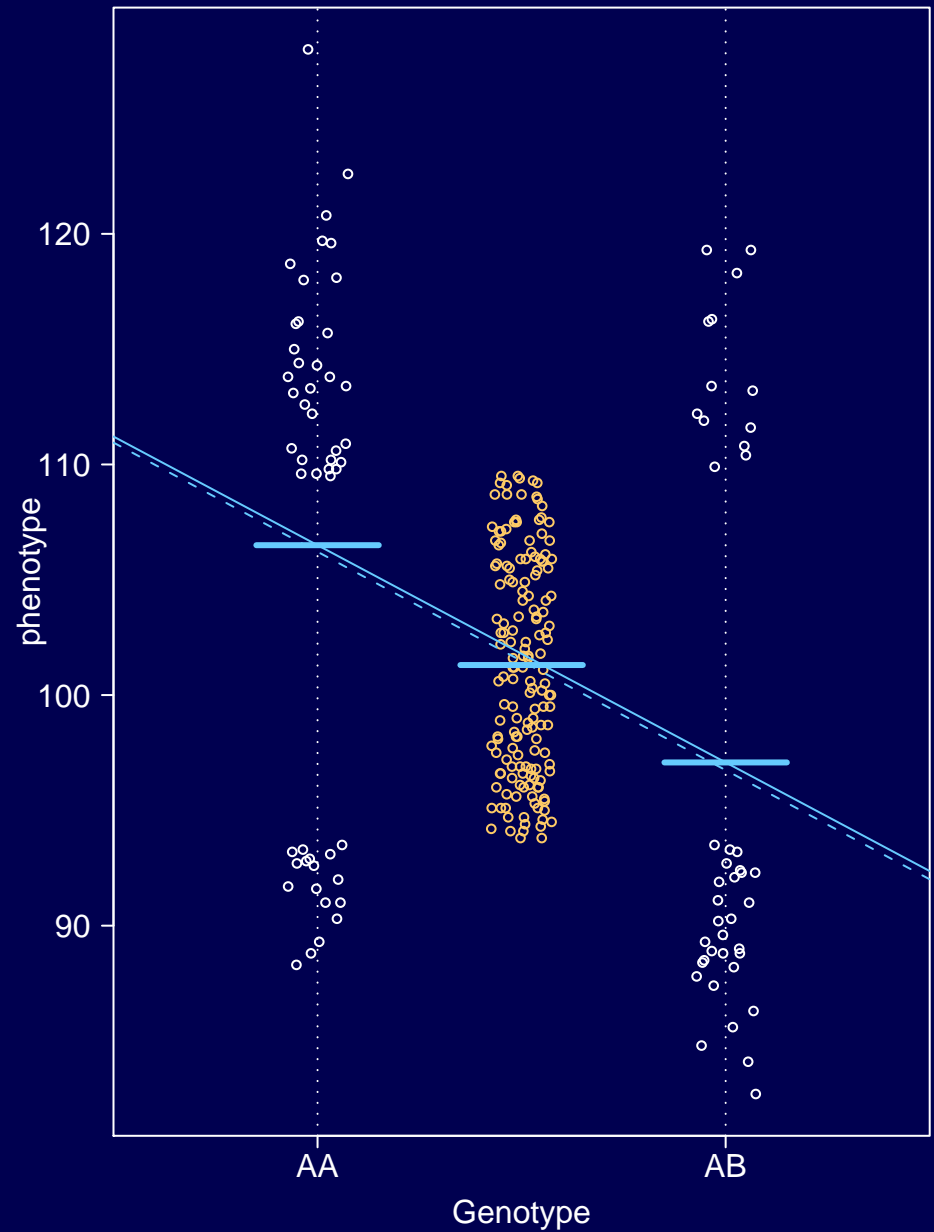
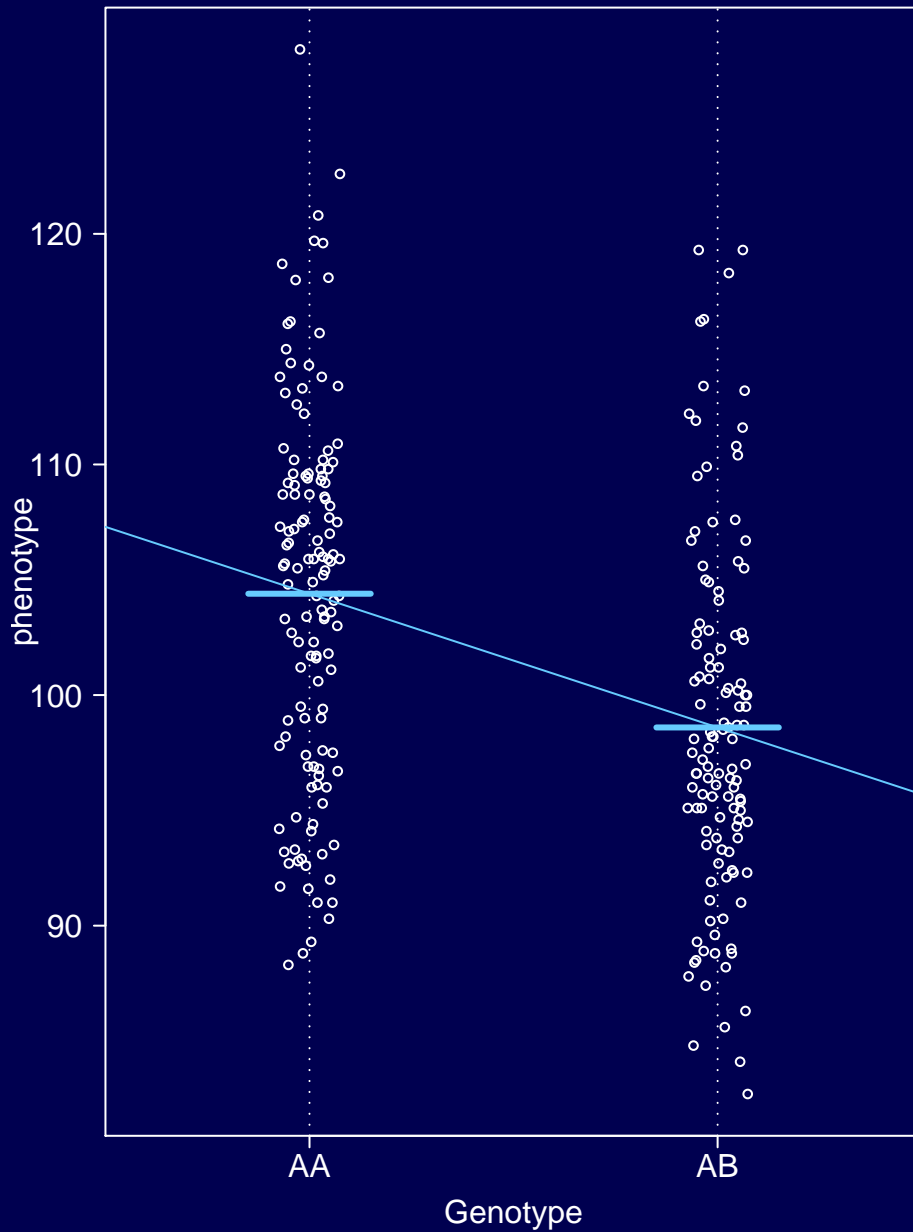
- Two markers separated by 20 cM, with the QTL closer to the left marker.
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Haley-Knott results



H-K with selective genotyping



[R/qtI]

LOD thresholds

Large LOD scores indicate evidence for the presence of a QTL

Question: How large is large?

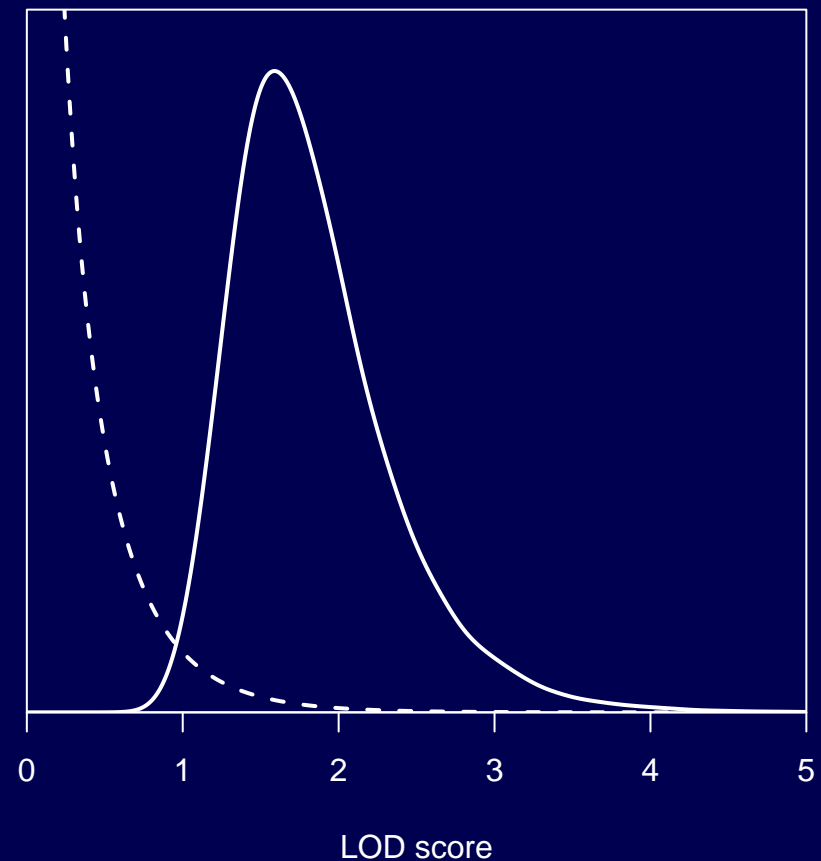
LOD threshold = 95 %ile of distr'n of max LOD, genome-wide, if there are no QTLs anywhere

Derivation:

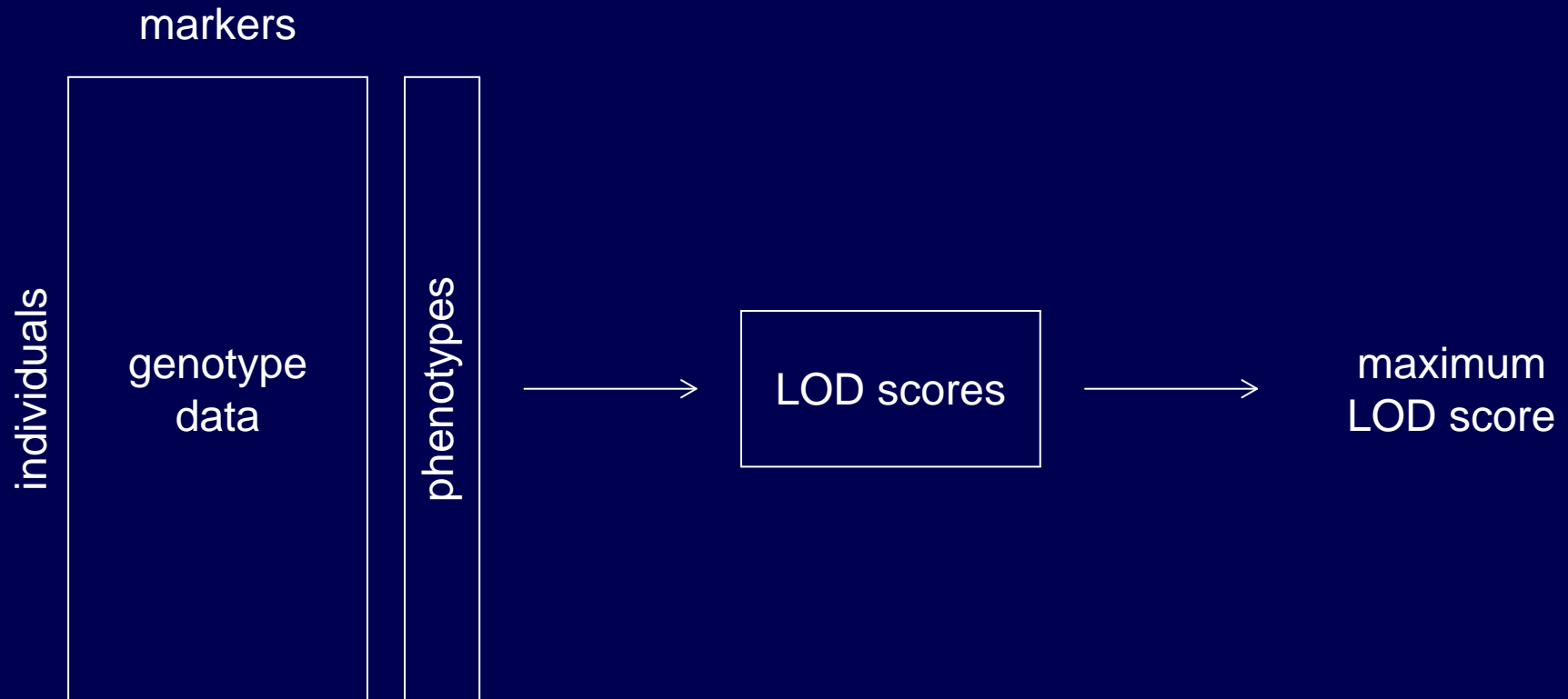
- Analytical calculations (L & B 1989)
- Simulations (L & B 1989)
- Permutation tests (Churchill & Doerge 1994)

Null distribution of the LOD score

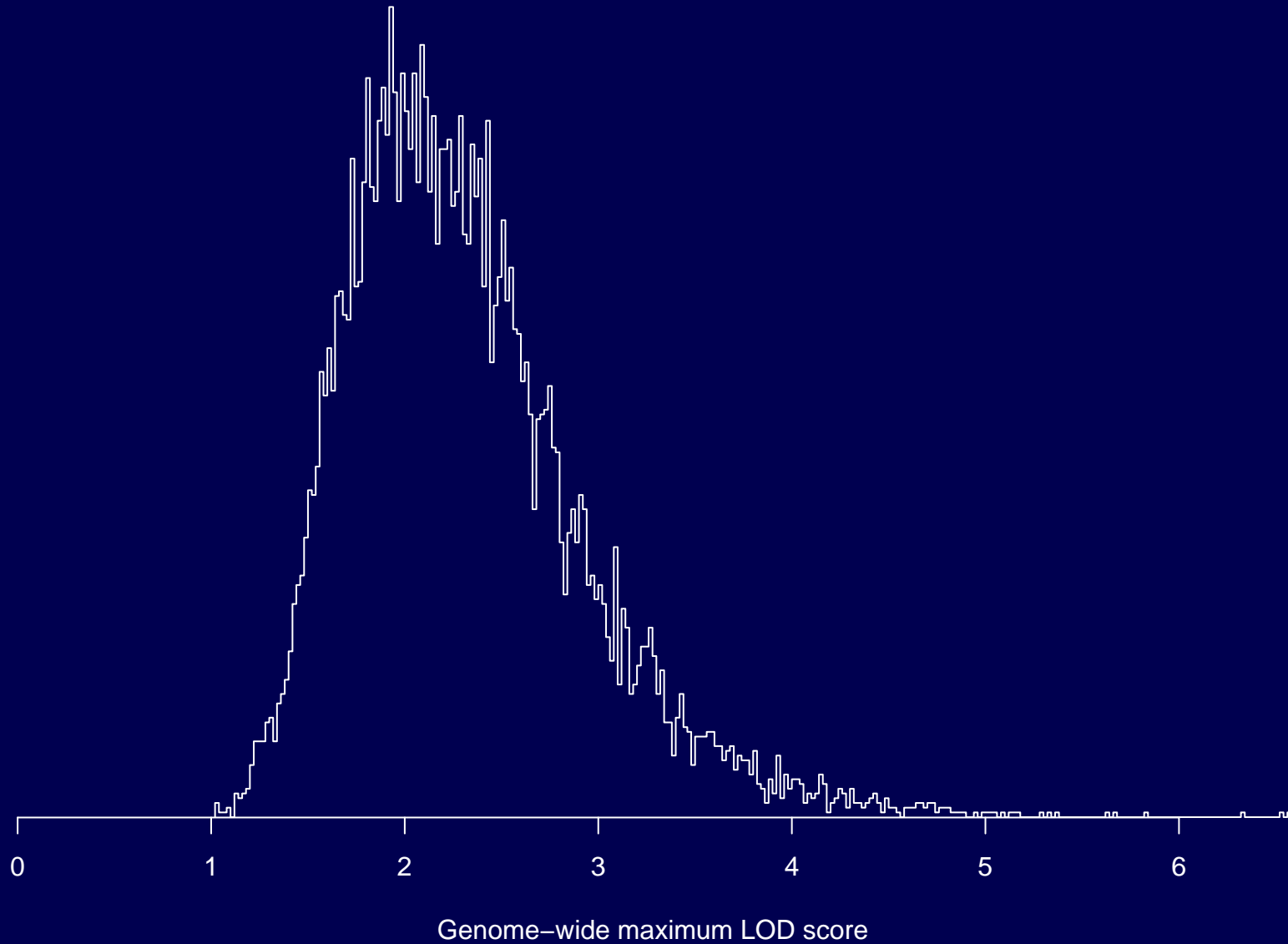
- Null distribution derived by computer simulation of backcross with genome of typical size.
- Dashed curve: distribution of LOD score at any one point.
- Solid curve: distribution of maximum LOD score, genome-wide.



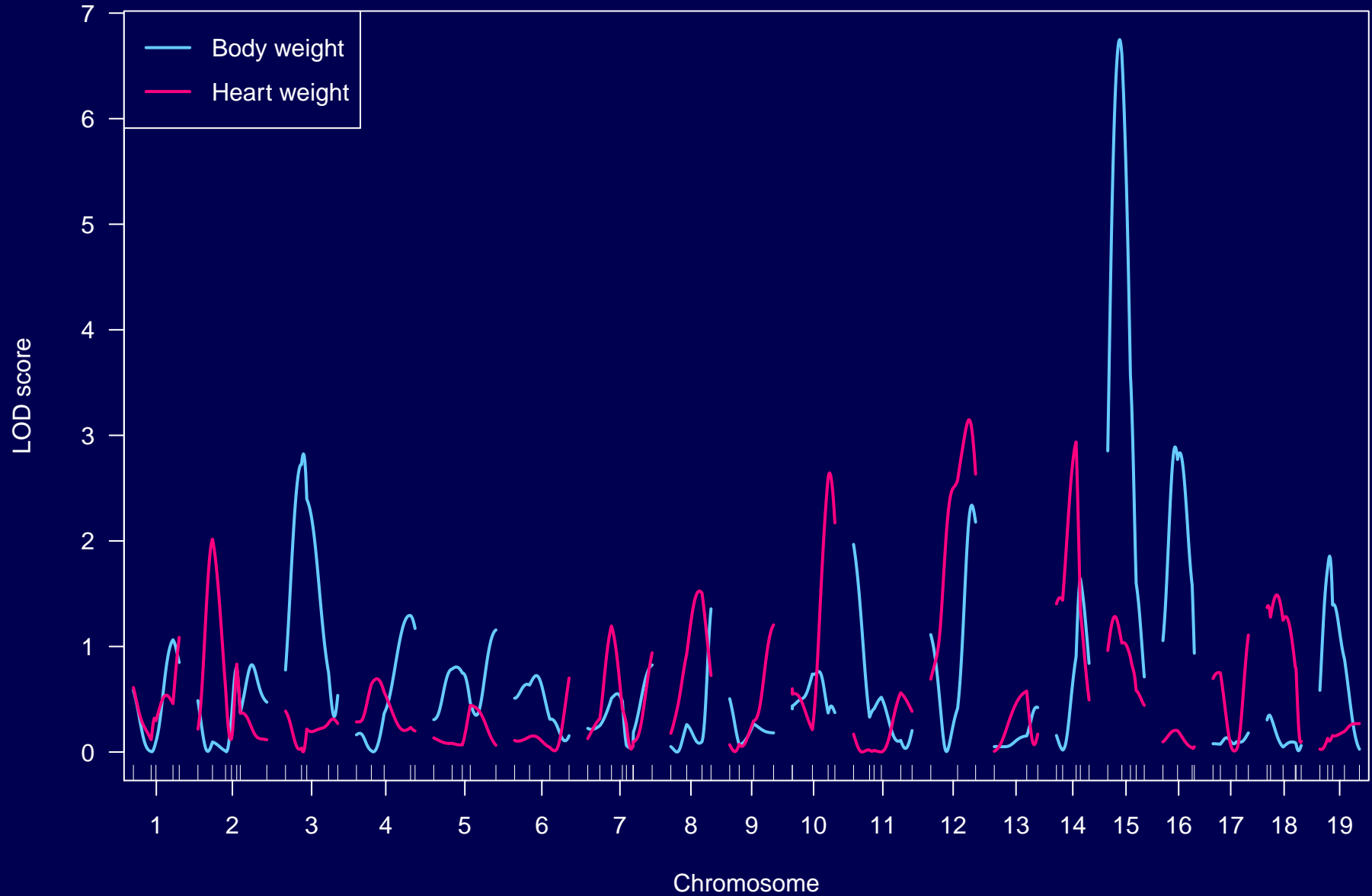
Permutation test



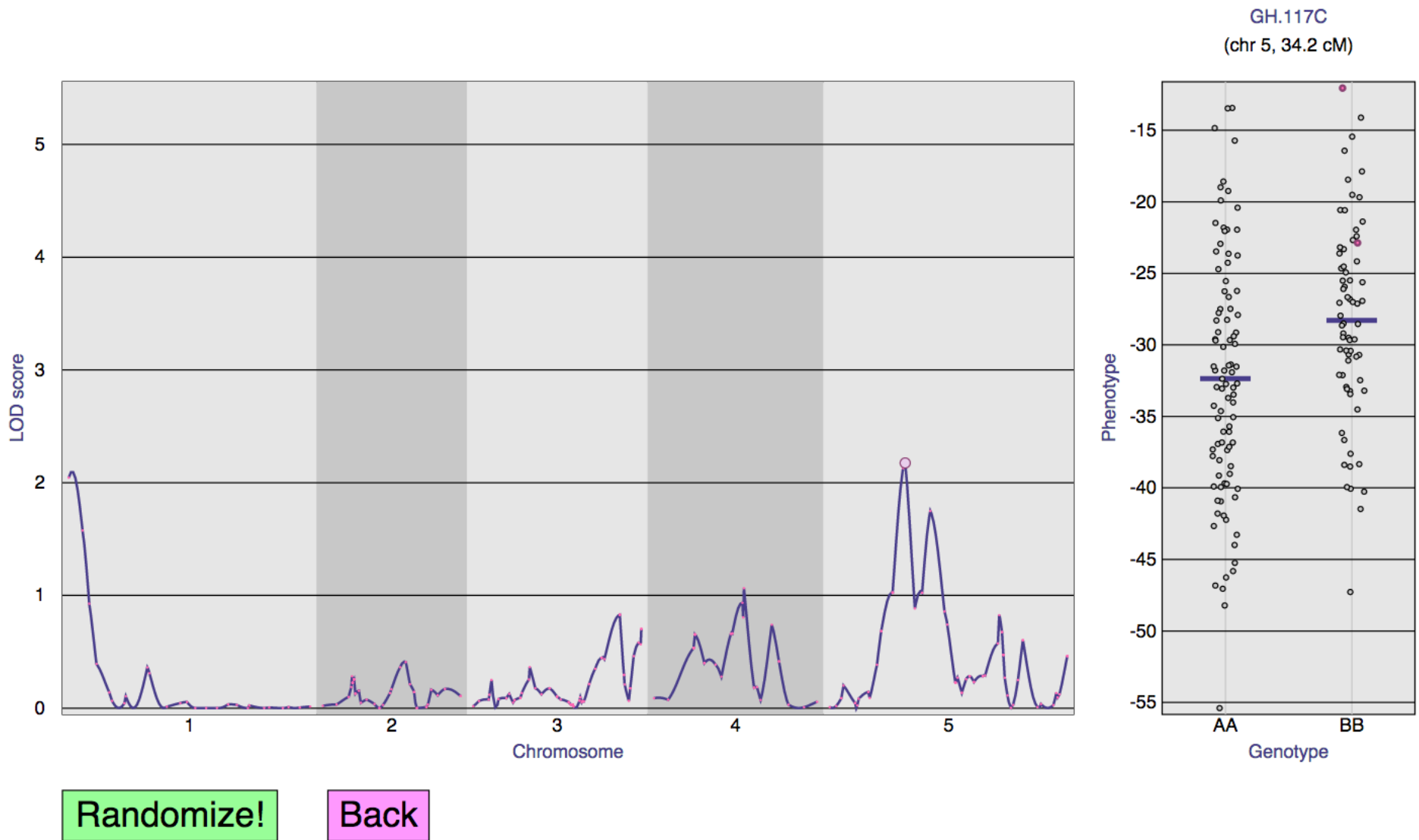
Permutation results



LOD curves

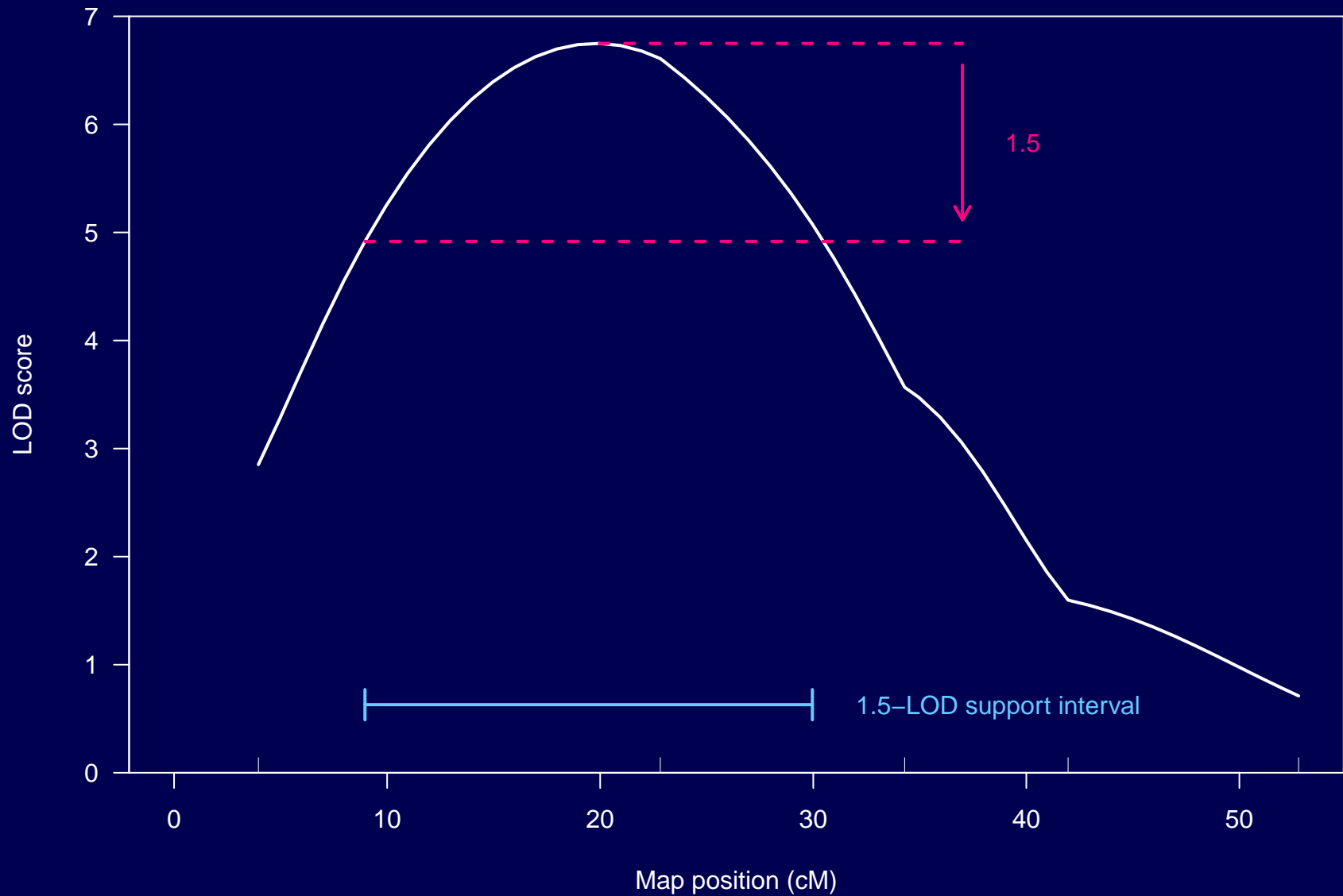


Interactive plot



[R/ql]

LOD support intervals



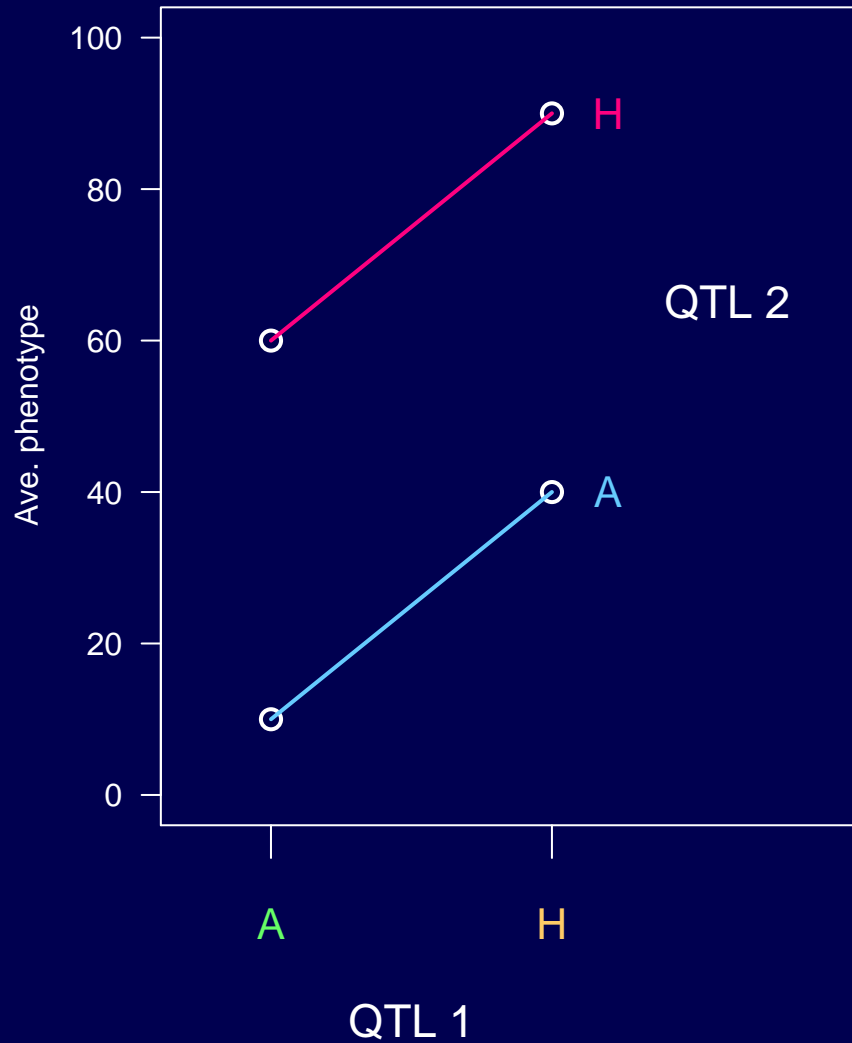
[R/qtI]

Modelling multiple QTL

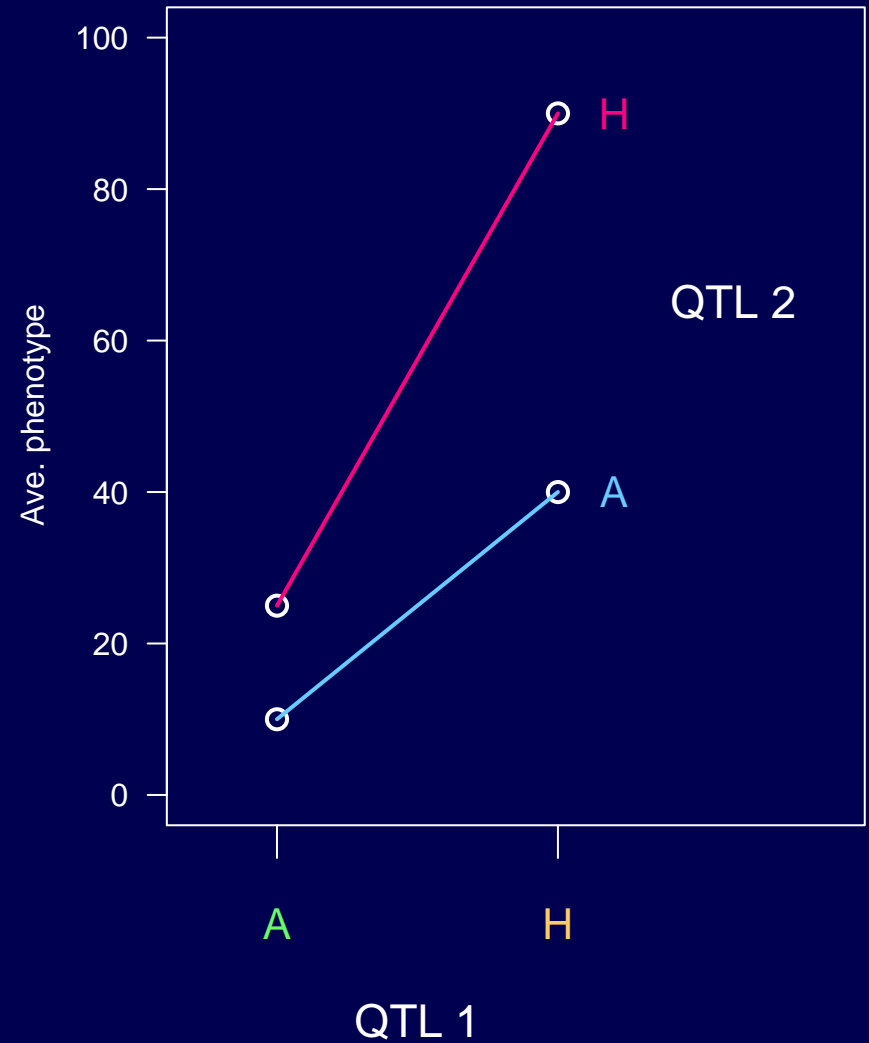
- Reduce residual variation \implies increased power
- Separate linked QTL
- Identify interactions among QTL

Epistasis in BC

Additive

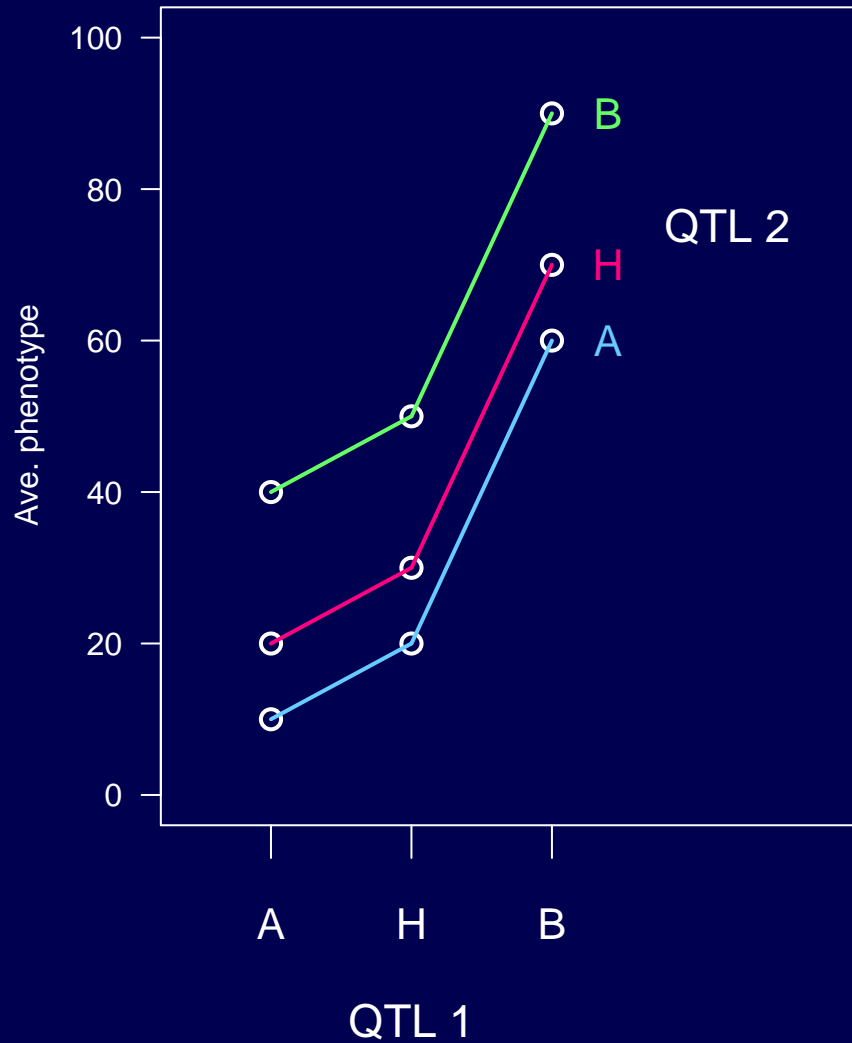


Epistatic

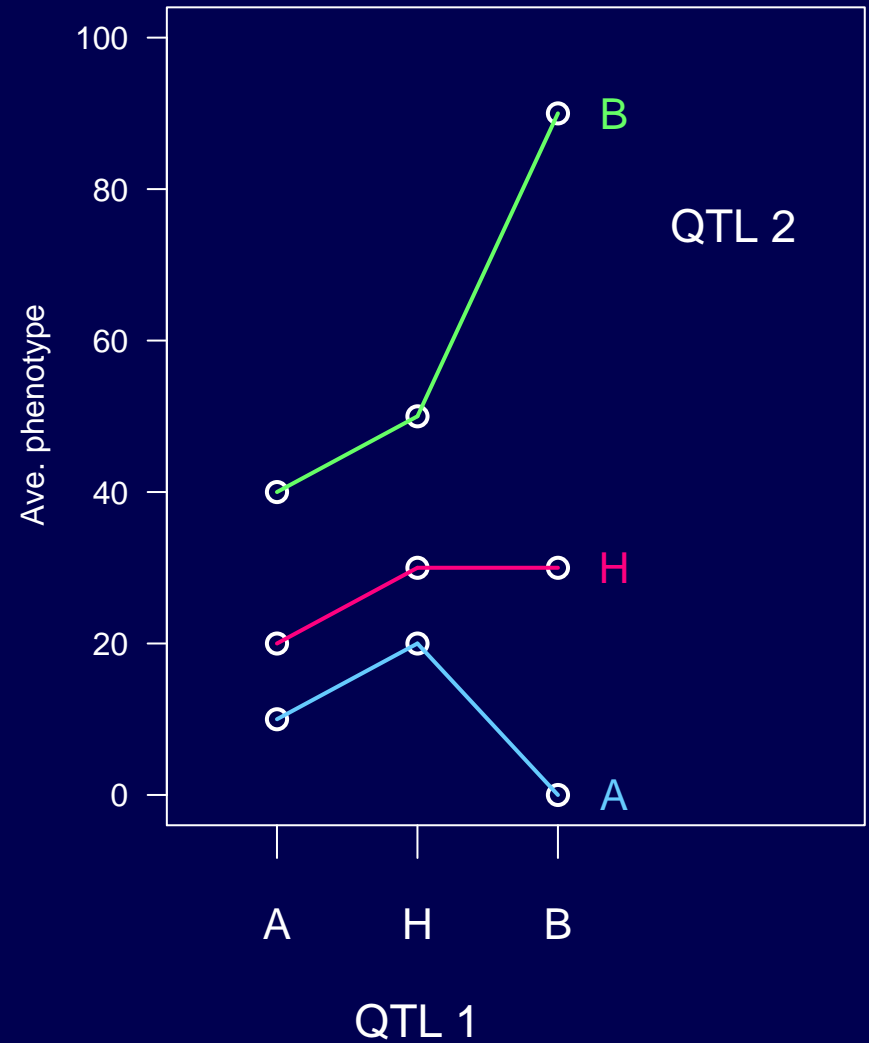


Epistasis in F₂

Additive



Epistatic



References

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A review for non-statisticians.
- Lynch M, Walsh B (1998) *Genetics and analysis of quantitative traits*. Sinauer Associates, Sunderland, MA, chapter 15
Chapter on QTL mapping.
- Lander ES, Botstein D (1989) Mapping Mendelian factors underlying quantitative traits using RFLP linkage maps. *Genetics* 121:185–199
The seminal paper.
- Churchill GA, Doerge RW (1994) Empirical threshold values for quantitative trait mapping. *Genetics* 138:963–971
LOD thresholds by permutation tests.
- Strickberger MW (1985) *Genetics*, 3rd edition. Macmillan, New York, chapter 11.
An old but excellent general genetics textbook with a very interesting discussion of epistasis.