

# Introduction to QTL mapping in model organisms

---

Karl W Broman

Department of Biostatistics  
Johns Hopkins University

`kbroman@jhsph.edu`

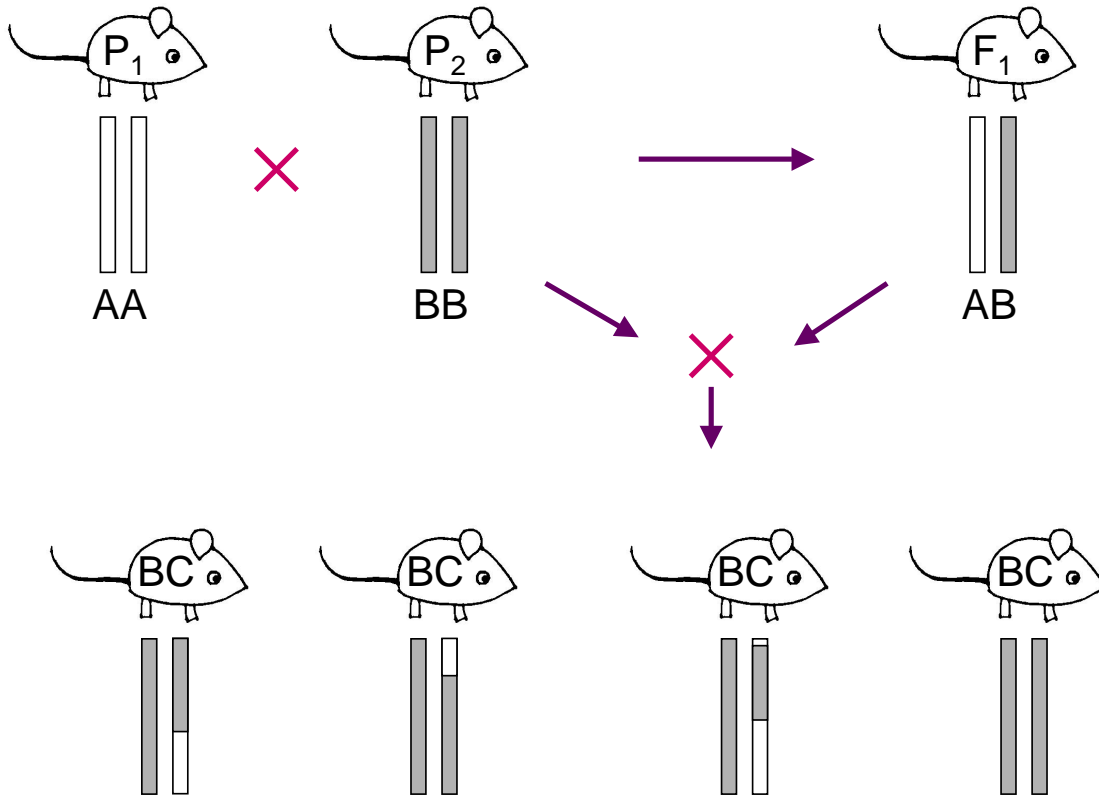
`www.biostat.jhsph.edu/~kbroman`

## Outline

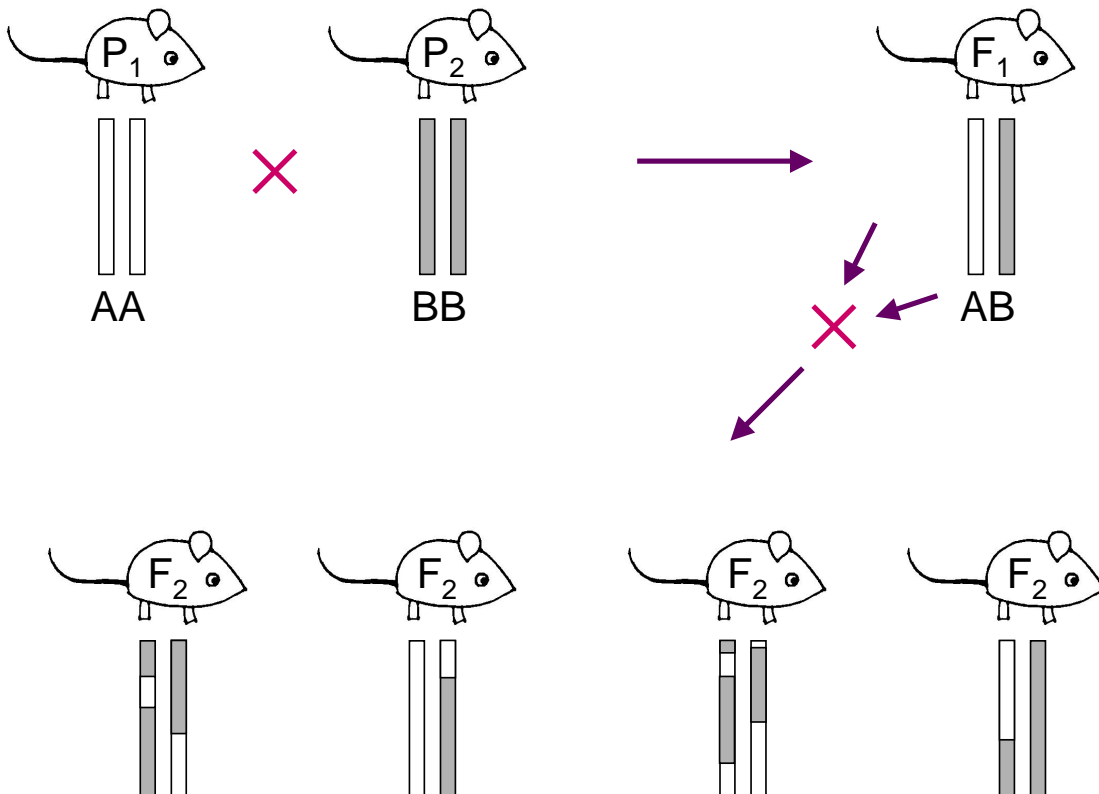
---

- Experiments and data
- Models
- ANOVA at marker loci
- Interval mapping
- Epistasis
- LOD thresholds
- CIs for QTL location
- Selection bias
- The X chromosome
- Selective genotyping
- Covariates
- Non-normal traits
- The need for good data

# Backcross experiment



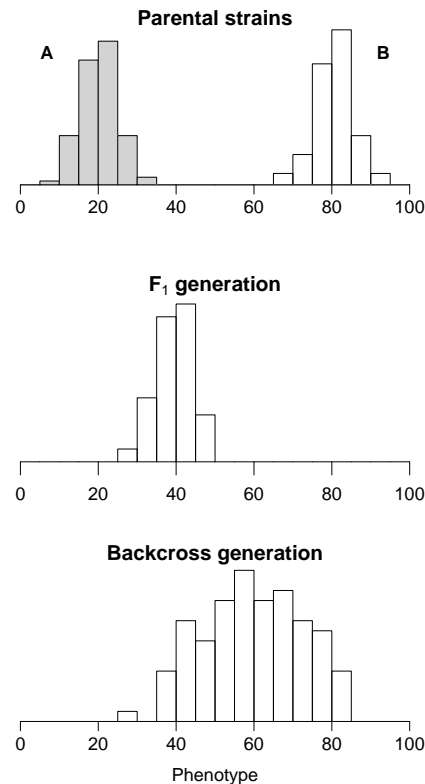
# Intercross experiment



# Phenotype distributions

---

- Within each of the parental and  $F_1$  strains, individuals are genetically identical.
- Environmental variation may or may not be constant with genotype.
- The backcross generation exhibits genetic as well as environmental variation.



## Data

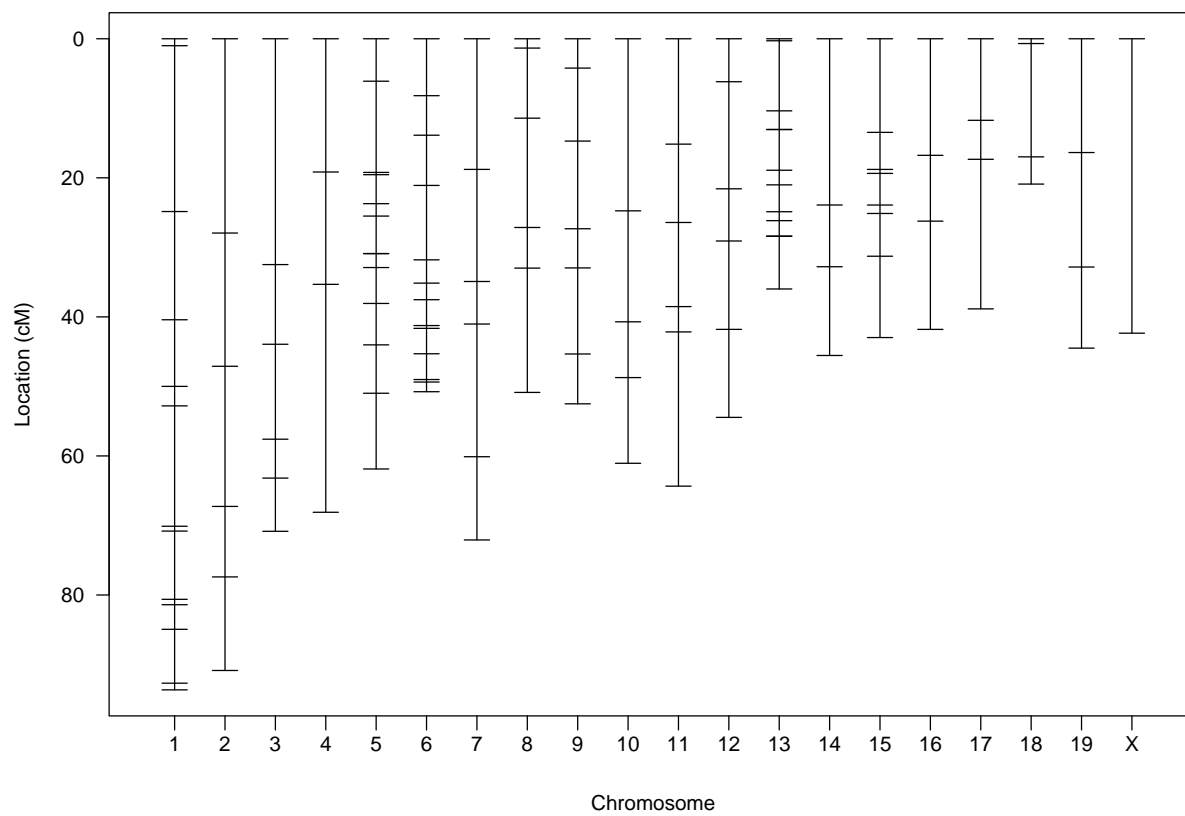
---

**Phenotypes:**  $y_i$  = trait value for individual  $i$

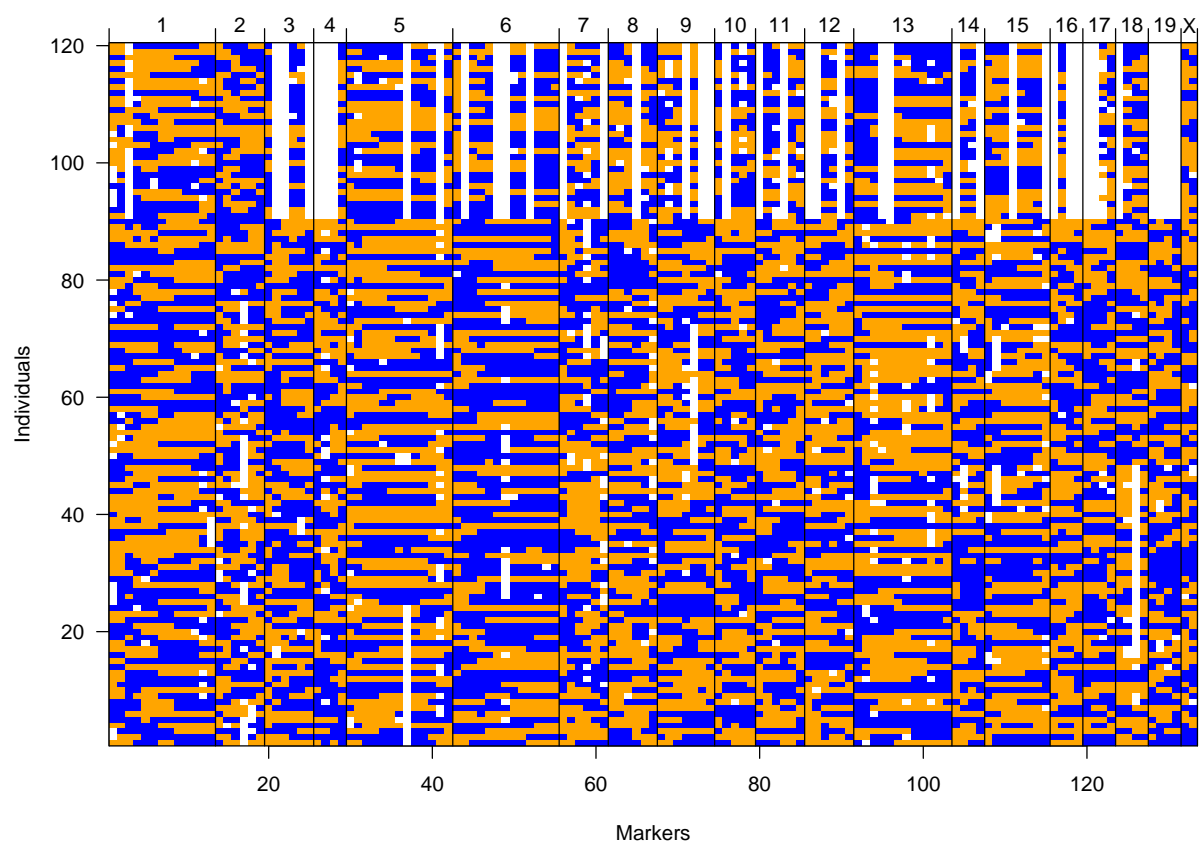
**Genotypes:**  $x_{ij}$  = 0/1 if mouse  $i$  is BB/AB at marker  $j$   
(or 0/1/2, in an intercross)

**Genetic map:** Locations of markers

## Genetic map



## Genotype data



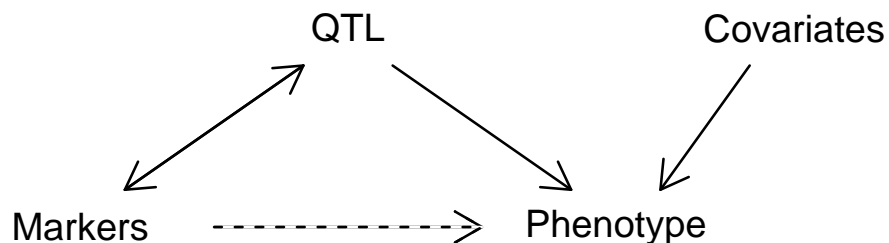
# Goals

---

- Detect QTLs (and interactions between QTLs)
- Confidence intervals for QTL location
- Estimate QTL effects (effects of allelic substitution)

## Statistical structure

---



The missing data problem:

Markers  $\longleftrightarrow$  QTL

The model selection problem:

QTL, covariates  $\longrightarrow$  phenotype

# Models: Recombination

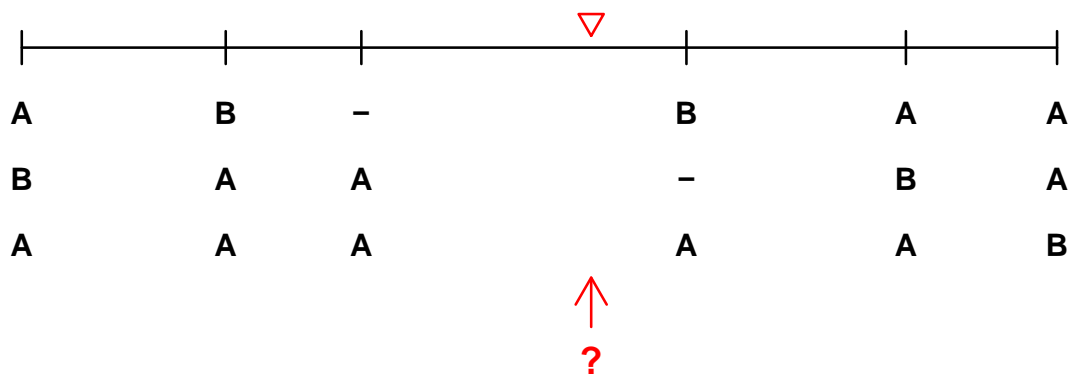
---

- We assume
- Mendel's rules
  - no crossover interference

1.  $\Pr(x_{ij} = 0) = \Pr(x_{ij} = 1) = 1/2$
2.  $\Pr(x_{i,j+1} = 1 \mid x_{ij} = 0) = \Pr(x_{i,j+1} = 0 \mid x_{ij} = 1) = r_j$   
where  $r_i$  is the recombination fraction for the interval
3. The genotype at a position depends only on the genotypes at the nearest flanking typed markers

## Example

---



## Models: Genotype $\longleftrightarrow$ Phenotype

---

Let  $y$  = phenotype  
 $g$  = whole genome genotype

Imagine a small number of QTLs with genotypes  $g_1, \dots, g_p$ .  
( $2^p$  distinct genotypes)

$$E(y|g) = \mu_{g_1, \dots, g_p} \quad \text{var}(y|g) = \sigma_{g_1, \dots, g_p}^2$$

## Models: Genotype $\longleftrightarrow$ Phenotype

---

**Homoscedasticity** (constant variance):  $\sigma_g^2 \equiv \sigma^2$

**Normally distributed residual variation:**  $y|g \sim N(\mu_g, \sigma^2)$ .

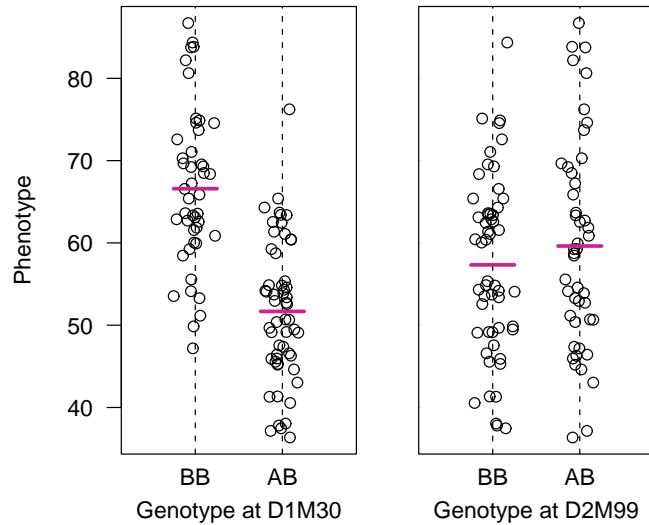
**Additivity:**  $\mu_{g_1, \dots, g_p} = \mu + \sum_{j=1}^p \Delta_j g_j$  ( $g_j = 1$  or  $0$ )

**Epistasis:** Any deviations from additivity.

# The simplest method: ANOVA

---

- Also known as **marker regression**.
- Split mice into groups according to genotype at a marker.
- Do a t-test / ANOVA.
- Repeat for each marker.



## ANOVA at marker loci

---

### Advantages

- Simple.
- Easily incorporates covariates.
- Easily extended to more complex models.
- Doesn't require a genetic map.

### Disadvantages

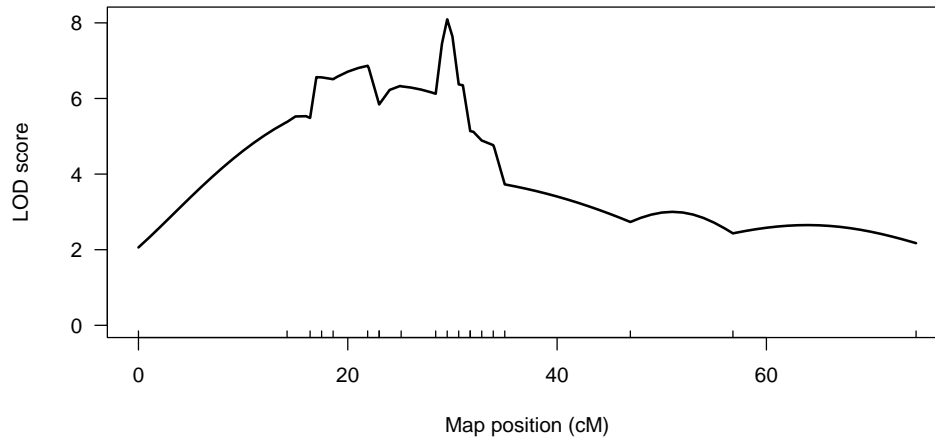
- Must exclude individuals with missing genotype data.
- Imperfect information about QTL location.
- Suffers in low density scans.
- Only considers one QTL at a time.



# Interval mapping (IM)

## Lander & Botstein (1989)

- Take account of missing genotype data
- Interpolate between markers
- Maximum likelihood under a mixture model



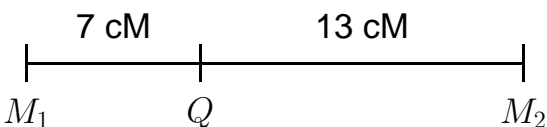
# Interval mapping (IM)

## Lander & Botstein (1989)

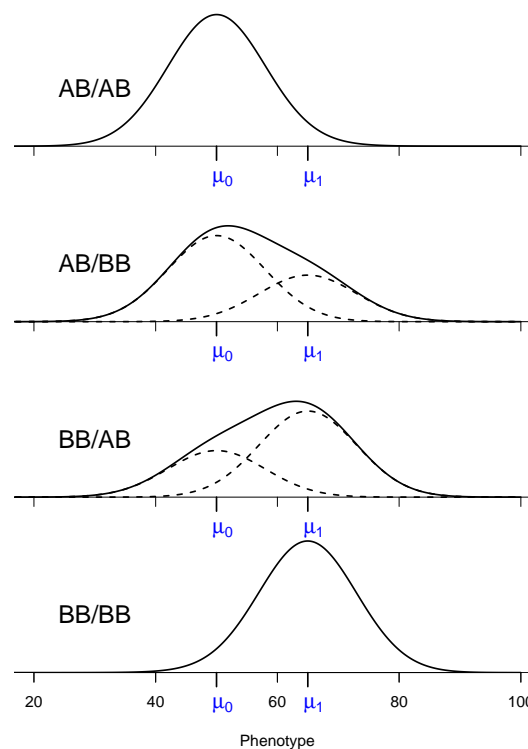
- Assume a **single** QTL model.
- Each position in the genome, one at a time, is posited as the putative QTL.
- Let  $z = 1/0$  if the (unobserved) QTL genotype is BB/AB.  
Assume  $y \sim N(\mu_z, \sigma)$
- Given genotypes at linked markers,  $y \sim$  mixture of normal dist'ns with mixing proportion  $\Pr(z = 1 | \text{marker data})$ :

		QTL genotype	
		BB	AB
$M_1$	$M_2$		
BB	BB	$(1 - r_L)(1 - r_R)/(1 - r)$	$r_L r_R/(1 - r)$
BB	AB	$(1 - r_L)r_R/r$	$r_L(1 - r_R)/r$
AB	BB	$r_L(1 - r_R)/r$	$(1 - r_L)r_R/r$
AB	AB	$r_L r_R/(1 - r)$	$(1 - r_L)(1 - r_R)/(1 - r)$

# The normal mixtures



- Two markers separated by 20 cM, with the QTL closer to the left marker.
- The figure at right show the distributions of the phenotype conditional on the genotypes at the two markers.
- The dashed curves correspond to the components of the mixtures.



## Interval mapping (continued)

Let  $p_i = \Pr(z_i = 1 | \text{marker data})$

$$y_i | z_i \sim N(\mu_{z_i}, \sigma^2)$$

$$\Pr(y_i | \text{marker data}, \mu_0, \mu_1, \sigma) = p_i f(y_i; \mu_1, \sigma) + (1 - p_i) f(y_i; \mu_0, \sigma)$$

where  $f(y; \mu, \sigma) = \text{density of normal distribution}$

**Log likelihood:**  $l(\mu_0, \mu_1, \sigma) = \sum_i \log \Pr(y_i | \text{marker data}, \mu_0, \mu_1, \sigma)$

Maximum likelihood estimates (**MLEs**) of  $\mu_0, \mu_1, \sigma$ :

values for which  $l(\mu_0, \mu_1, \sigma)$  is maximized.

# LOD scores

---

The LOD score is a measure of the **strength of evidence** for the presence of a QTL at a particular location.

$\text{LOD}(z) = \log_{10}$  likelihood ratio comparing the hypothesis of a QTL at position  $z$  versus that of no QTL

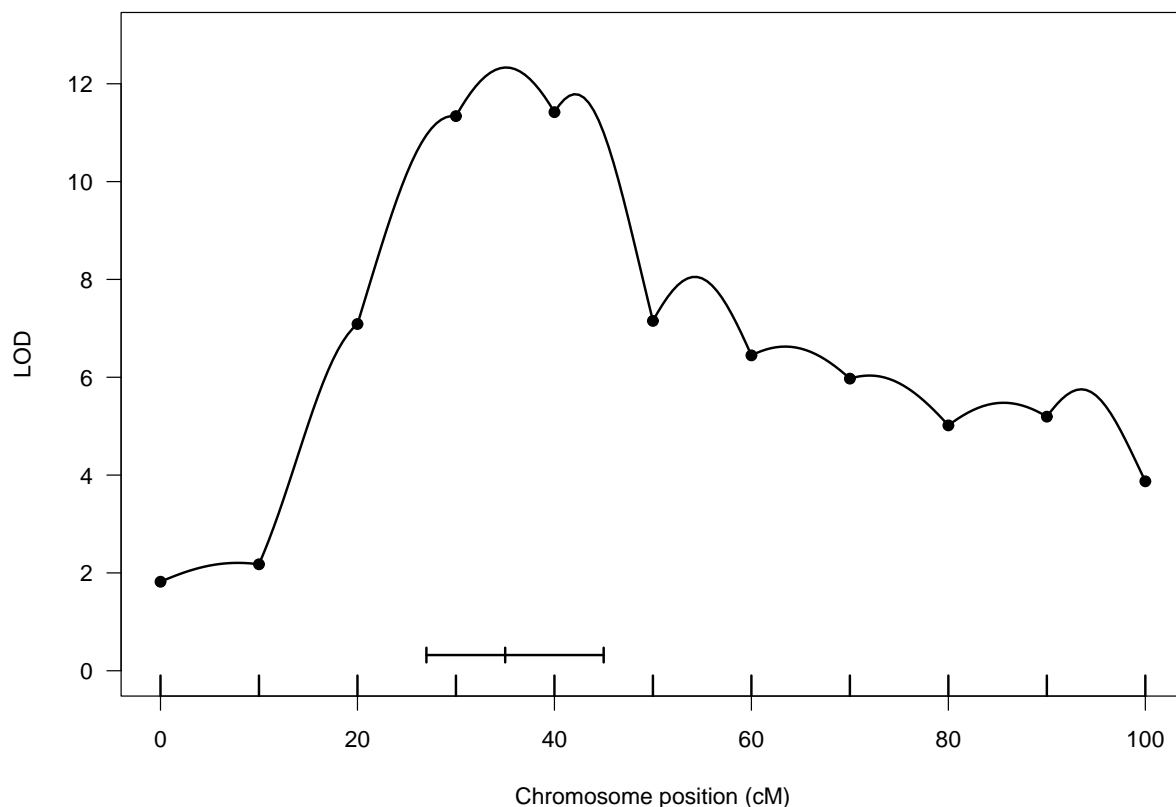
$$= \log_{10} \left\{ \frac{\Pr(y|\text{QTL at } z, \hat{\mu}_{0z}, \hat{\mu}_{1z}, \hat{\sigma}_z)}{\Pr(y|\text{no QTL}, \hat{\mu}, \hat{\sigma})} \right\}$$

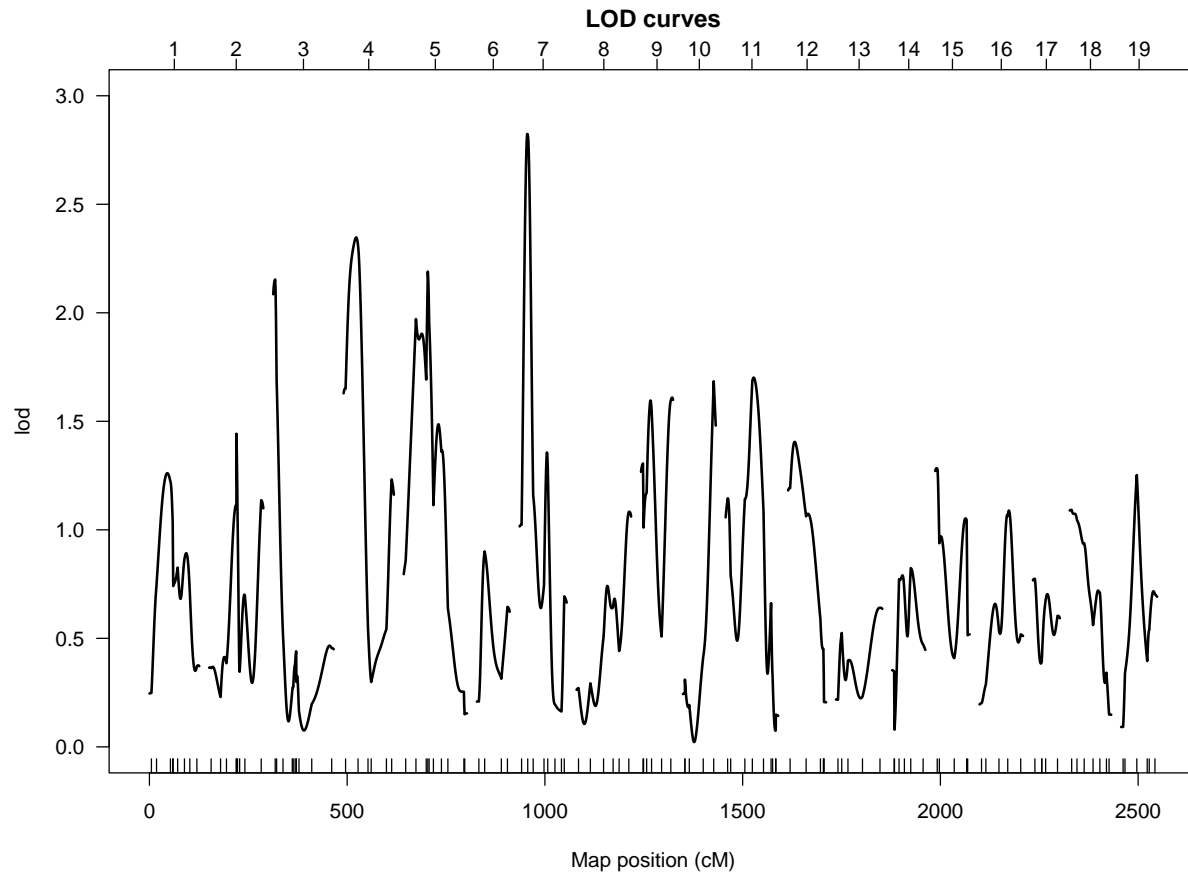
$\hat{\mu}_{0z}, \hat{\mu}_{1z}, \hat{\sigma}_z$  are the MLEs, assuming a single QTL at position  $z$ .

No QTL model: The phenotypes are independent and identically distributed (iid)  $N(\mu, \sigma^2)$ .

## An example LOD curve

---





## Interval mapping

---

### Advantages

- Takes proper account of missing data.
- Allows examination of positions between markers.
- Gives improved estimates of QTL effects.
- Provides pretty graphs.

### Disadvantages

- Increased computation time.
- Requires specialized software.
- Difficult to generalize.
- Only considers one QTL at a time.

# Multiple QTL methods

---

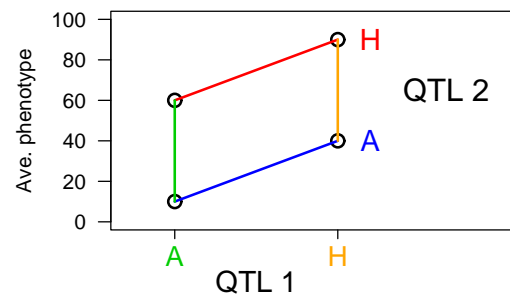
## Why consider multiple QTLs at once?

- Reduce residual variation.
- Separate linked QTLs.
- Investigate interactions between QTLs (epistasis).

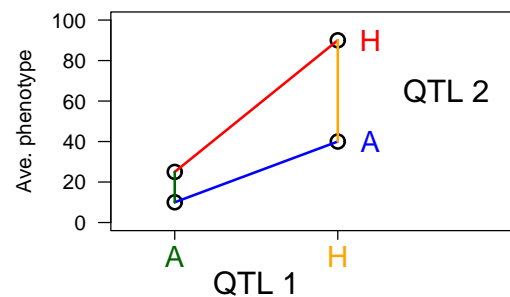
## Epistasis in a backcross

---

Additive QTLs



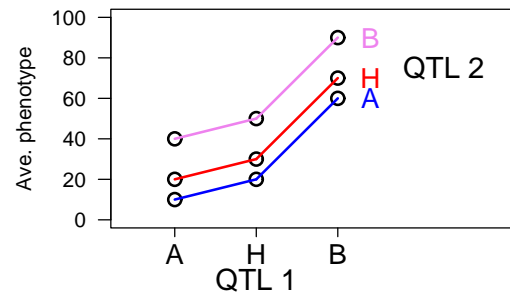
Interacting QTLs



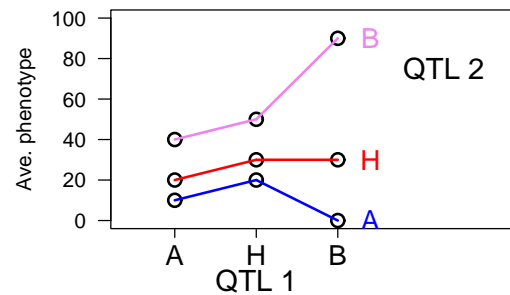
# Epistasis in an intercross

---

Additive QTLs



Interacting QTLs



## LOD thresholds

---

Large LOD scores indicate evidence for the presence of a QTL.

**Q:** How large is large?

→ We consider the distribution of the LOD score under the null hypothesis of no QTL.

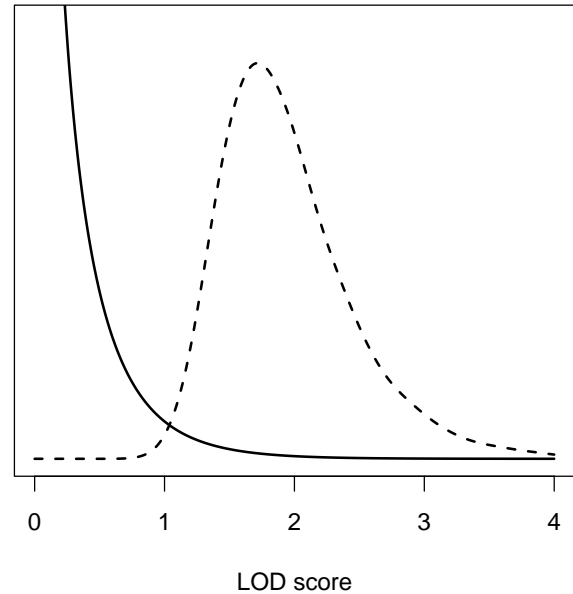
**Key point:** We must make some adjustment for our examination of multiple putative QTL locations.

→ We seek the distribution of the *maximum* LOD score, genome-wide. The 95th %ile of this distribution serves as a **genome-wide LOD threshold**.

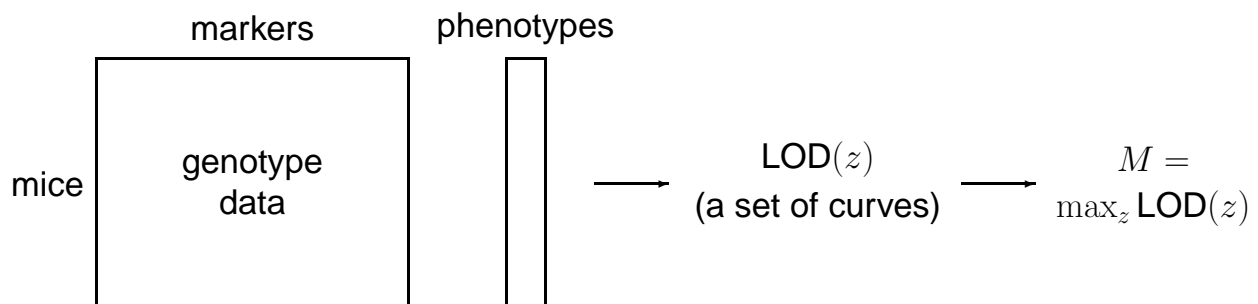
Estimating the threshold: simulations, analytical calculations, permutation (randomization) tests.

# Null distribution of the LOD score

- Null distribution derived by computer simulation of backcross with genome of typical size.
- Solid curve: distribution of LOD score at any one point.
- Dashed curve: distribution of maximum LOD score, genome-wide.



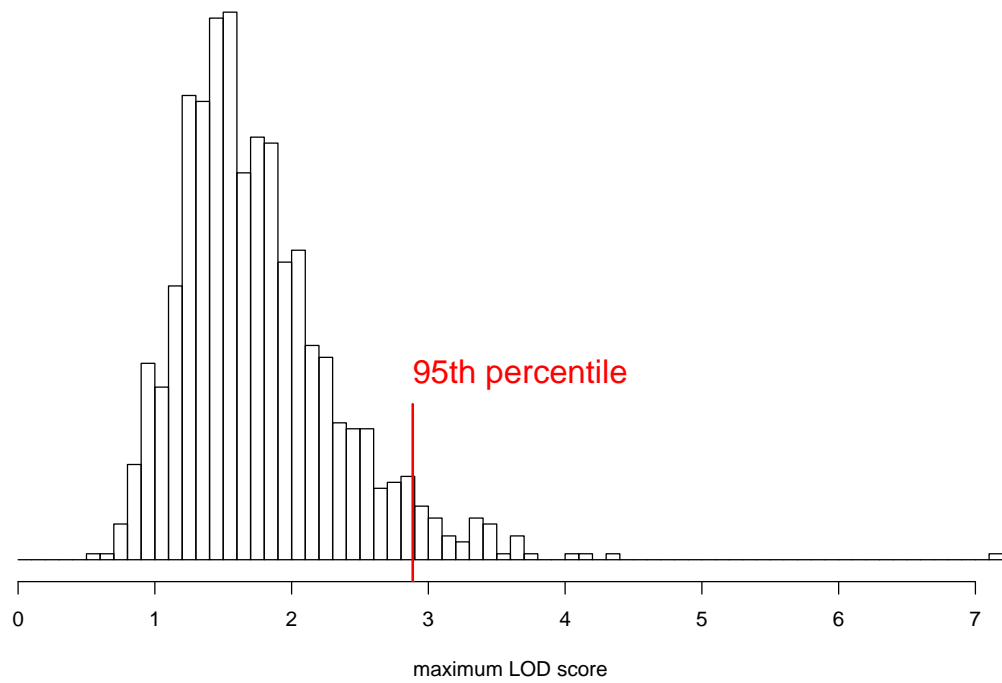
## Permutation tests



- Permute/shuffle the phenotypes; keep the genotype data intact.
- Calculate  $\text{LOD}^*(z) \rightarrow M^* = \max_z \text{LOD}^*(z)$
- We wish to compare the observed  $M$  to the distribution of  $M^*$ .
- $\Pr(M^* \geq M)$  is a genome-wide P-value.
- The 95th %ile of  $M^*$  is a genome-wide LOD threshold.
- We can't look at all  $n!$  possible permutations, but a random set of 1000 is feasible and provides reasonable estimates of P-values and thresholds.
- **Value:** conditions on observed phenotypes, marker density, and pattern of missing data; doesn't rely on normality assumptions or asymptotics.

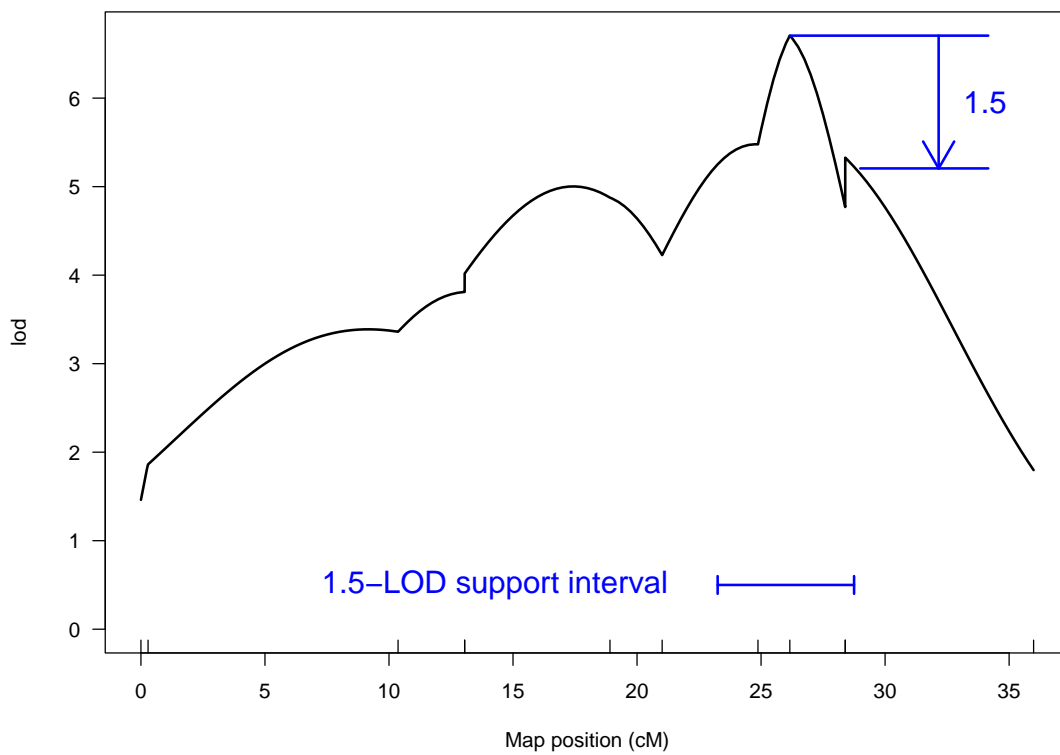
# Permutation distribution

---



# 1.5-LOD support interval

---

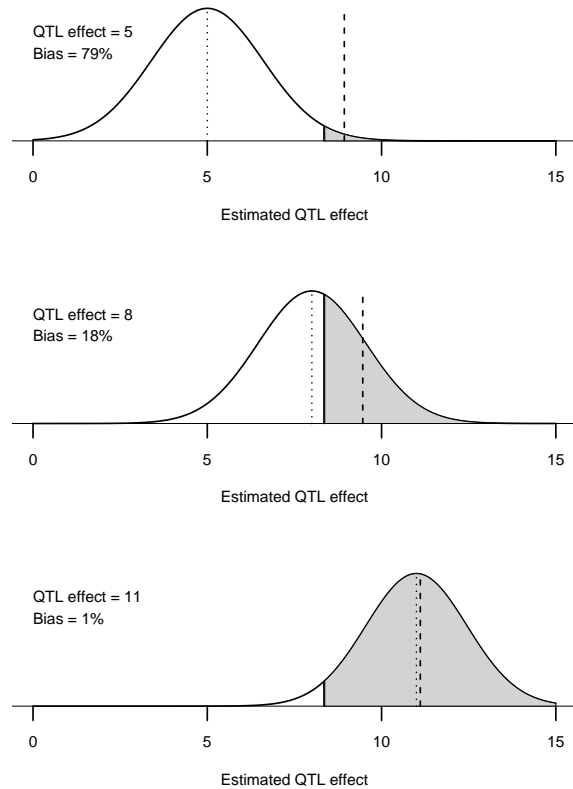




# Selection bias

---

- The estimated effect of a QTL will vary somewhat from its true effect.
- Only when the estimated effect is large will the QTL be detected.
- Among those experiments in which the QTL is detected, the estimated QTL effect will be, on average, larger than its true effect.
- This is **selection bias**.
- Selection bias is largest in QTLs with small or moderate effects.
- The true effects of QTLs that we identify are likely smaller than was observed.



## Implications of selection bias

---

- Estimated % variance explained by identified QTLs
- Repeating an experiment
- Congenics
- Marker-assisted selection

## The X chromosome

---

In a backcross, the X chromosome may or may not be segregating.

$$(A \times B) \times A$$

Females:  $X_{A \cdot B} X_A$

Males:  $X_{A \cdot B} Y_A$

$$A \times (A \times B)$$

Females:  $X_A X_A$

Males:  $X_A Y_B$

## The X chromosome

---

In an intercross, one must pay attention to the **paternal grandmother's genotype**.

$$(A \times B) \times (A \times B) \quad \text{or} \quad (B \times A) \times (A \times B)$$

Females:  $X_{A \cdot B} X_A$

Males:  $X_{A \cdot B} Y_B$

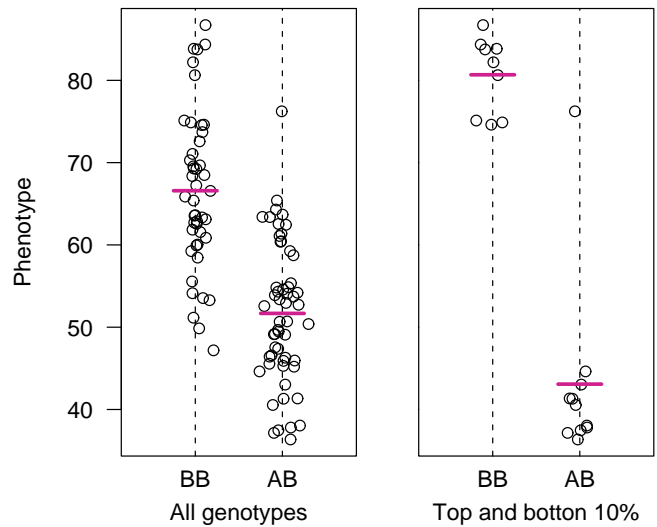
$$(A \times B) \times (B \times A) \quad \text{or} \quad (B \times A) \times (B \times A)$$

Females:  $X_{A \cdot B} X_B$

Males:  $X_{A \cdot B} Y_A$

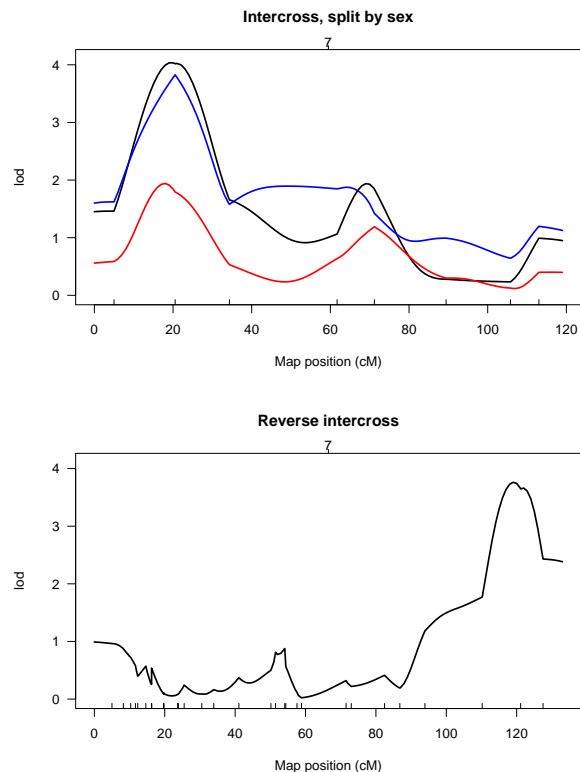
# Selective genotyping

- Save effort by only typing the most informative individuals (say, top & bottom 10%).
- Useful in context of a **single, inexpensive** trait.
- Tricky to estimate the effects of QTLs: use IM with **all** phenotypes.
- Can't get at interactions.
- Likely better to also genotype some random portion of the rest of the individuals.



# Covariates

- **Examples:** treatment, sex, litter, lab, age.
- Control residual variation.
- Avoid confounding.
- Look for QTL  $\times$  environ't interactions
- Adjust before interval mapping (IM) versus adjust within IM.



## Non-normal traits

---

- Standard interval mapping assumes normally distributed residual variation. (Thus the phenotype distribution is a mixture of normals.)
- **In reality:** we see dichotomous traits, counts, skewed distributions, outliers, and all sorts of odd things.
- Interval mapping, with LOD thresholds derived from permutation tests, generally performs just fine anyway.
- Alternatives to consider:
  - Nonparametric approaches (Kruglyak & Lander 1995)
  - Transformations (e.g., log, square root)
  - Specially-tailored models (e.g., a generalized linear model, the Cox proportional hazard model, and the model in Broman et al. 2000)

## Check data integrity

---

The success of QTL mapping depends crucially on the integrity of the data.

- Segregation distortion
- Genetic maps / marker positions
- Genotyping errors (tight double crossovers)
- Phenotype distribution / outliers
- Residual analysis

# Summary I

---

- **ANOVA** at marker loci (aka marker regression) is simple and easily extended to include covariates or accommodate complex models.
- **Interval mapping** improves on ANOVA by allowing inference of QTLs to positions between markers and taking proper account of missing genotype data.
- ANOVA and IM consider only single-QTL models. **Multiple QTL methods** allow the better separation of linked QTLs and are necessary for the investigation of epistasis.
- Statistical significance of LOD peaks requires consideration of the maximum LOD score, genome-wide, under the null hypothesis of no QTLs. **Permutation tests** are extremely useful for this.
- **1.5-LOD support intervals** indicate the plausible location of a QTL.
- Estimates of QTL effects are subject to **selection bias**. Such estimated effects are often too large.

# Summary II

---

- The **X chromosome** must be dealt with specially, and can be tricky.
- **Study your data**. Look for errors in the genetic map, genotyping errors and phenotype outliers. But don't worry about them too much.
- **Selective genotyping** can save you time and money, but proceed with caution.
- **Study your data**. The consideration of covariates may reveal extremely interesting phenomena.
- Interval mapping works reasonably well even with **non-normal traits**. But consider transformations or specially-tailored models. If interval mapping software is not available for your preferred model, start with some version of ANOVA.

# References

---

- Broman KW (2001) Review of statistical methods for QTL mapping in experimental crosses. *Lab Animal* 30(7):44–52  
[A review for non-statisticians.](#)
- Doerge RW (2002) Mapping and analysis of quantitative trait loci in experimental populations. *Nat Rev Genet* 3:43–52  
[A very recent review.](#)
- Doerge RW, Zeng Z-B, Weir BS (1997) Statistical issues in the search for genes affecting quantitative traits in experimental populations. *Statistical Science* 12:195–219  
[Review paper.](#)
- Jansen RC (2001) Quantitative trait loci in inbred lines. In Balding DJ et al., *Handbook of statistical genetics*, John Wiley & Sons, New York, chapter 21  
[Review in an expensive but rather comprehensive and likely useful book.](#)
- Lynch M, Walsh B (1998) *Genetics and analysis of quantitative traits*. Sinauer Associates, Sunderland, MA, chapter 15  
[Chapter on QTL mapping.](#)
  
- Lander ES, Botstein D (1989) Mapping Mendelian factors underlying quantitative traits using RFLP linkage maps. *Genetics* 121:185–199  
[The seminal paper.](#)
- Churchill GA, Doerge RW (1994) Empirical threshold values for quantitative trait mapping. *Genetics* 138:963–971  
[LOD thresholds by permutation tests.](#)
- Kruglyak L, Lander ES (1995) A nonparametric approach for mapping quantitative trait loci. *Genetics* 139:1421–1428  
[Non-parameteric interval mapping.](#)
- Boyartchuk VL, Broman KW, Mosher RE, D’Orazio S, Starnbach M, Dietrich WF (2001) Multigenic control of *Listeria monocytogenes* susceptibility in mice. *Nat Genet* 27:259–260
- Broman KW (2003) Mapping quantitative trait loci in the case of a spike in the phenotype distribution. *Genetics* 163:1169–1175  
[QTL mapping with a special model for a non-normal phenotype.](#)