1. Basic concepts
1.1 Data Structure
Assume we have k treatment groups
Group 1: $Y_{11}, Y_{12}, Y_{13}, Y_{14}, \ldots$

\[ \vdots \]

Group k: $Y_{k1}, Y_{k2}, Y_{k3}, Y_{k4}, \ldots$

1.2 Some Notations
- $n_t$: number of cases in treatment group t
- N: number of cases (overall)
- $Y_{it}$: response i in treatment group t
- $Y_t$: average response in treatment group t
- $\bar{Y}$: average response (overall)

1.3 Sum of Squares
1) Sum of squares for each group
   \[ S_t = \sum_{i=1}^{n_t} (Y_{it} - \bar{Y}_t)^2. \]

2) Within group sum of squares, df$_W$ = K-1
   \[ S_W = S_1 + \ldots + S_k = \sum_{t} \sum_{i} (Y_{it} - \bar{Y}_t)^2 \]

3) Between group sum of squares, df$_B$ = N-k
   \[ S_B = \sum_{t=1}^{k} \sum_{i} n_t (Y_{it} - \bar{Y}_t)^2 \]

4) Total group sum of squares: $S_T = S_W + S_B$, df = N-1 = df$_W$+df$_B$

1.4 ANOVA Table

<table>
<thead>
<tr>
<th>source</th>
<th>sum of squares</th>
<th>df</th>
<th>mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>between treatments</td>
<td>$S_B = \sum_{t} n_t (Y_{it} - \bar{Y}_t)^2$</td>
<td>k-1</td>
<td>$M_B = S_B/(k-1)$</td>
</tr>
<tr>
<td>within treatments</td>
<td>$S_W = \sum_{t} \sum_{i} (Y_{it} - \bar{Y}_t)^2$</td>
<td>N-k</td>
<td>$M_W = S_W/(N-k)$</td>
</tr>
<tr>
<td>total</td>
<td>$S_T = \sum_{t} \sum_{i} (Y_{it} - \bar{Y}_t)^2$</td>
<td>N-1</td>
<td></td>
</tr>
</tbody>
</table>

Note: book uses SS$_B$, SS$_W$, SS$_T$, MS$_B$, MS$_W$
2. Standard ANOVA Model

2.1 Model

\[ Y_{ti} = \mu_t + \varepsilon_{ti} \]
\[ Y_{ti} = \mu + \tau_t + \varepsilon_{ti}, \]
where \( Y_{ti} \sim \text{iid } N(\mu_t, \sigma^2) \) and \( \varepsilon_{ti} \sim \text{iid } N(0, \sigma^2) \)

2.2 Hypothesis Test

1) State the hypothesis
   \( H_0 : \mu_1 = \cdots = \mu_k \) vs. \( H_a : H_0 \text{ is false} \)

2) Test statistic
   \( F = \frac{M_B}{M_W} \)

3) Distribution of F under \( H_0 \)
   \( F \sim F_{k-1, N-k} \)
   where \( k-1 \) is the numerator degrees of freedom, \( N-k \) is the denominator degrees of freedom.

4) Use one-sided F test to find the p-value
   \( p\text{-value} = \Pr(F>F_{\text{obs}}) \)
   Method I: Table 10
   Method II. \( p\text{-value} = 1 - pf(F_{\text{obs}, k-1, N-k}) \)

3. Example

11.13 A researcher studied the flexibility of 10 women in an aerobic exercise class, 10 women in a modern dance class and a control group of 9 women. One measurement she made on each woman was spinal extension, which was a measure of how far the woman could bend her back. Measurements were made before and after a 16-week training period. The change in spinal extension was recorded for each woman. The ANOVA \( SS_B \) is 7.04 and the \( SS_W \) is 15.08.

(a) State the null hypothesis
(b) Construct the ANOVA table and test the null hypothesis. Let \( \alpha = 0.01 \)