Stat 371-003, Solutions to Homework #3

- **3.7 (pg 87)**

$$\begin{aligned}
\Pr(\text{affected}) &= \Pr(\text{affected and male}) + \Pr(\text{affected and female}) \\
&= \Pr(\text{male}) \times \Pr(\text{affected} \mid \text{male}) + \Pr(\text{female}) \times \Pr(\text{affected} \mid \text{female}) \\
&= 0.513 \times 0.5 + (1 - 0.513) \times 0 \\
&= 0.2565
\end{aligned}$$

- **3.9 (pg 88)**

  (a)

$$\begin{aligned}
\Pr(\text{positive}) &= \Pr(\text{positive and pregnant}) + \Pr(\text{positive and not pregnant}) \\
&= \Pr(\text{pregnant}) \times \Pr(\text{positive} \mid \text{pregnant}) + \Pr(\text{not pregnant}) \times \Pr(\text{positive} \mid \text{pregnant}) \\
&= 0.1 \times 0.98 + 0.9 \times (1 - 0.99) \\
&= 0.098 + 0.009 = 0.107
\end{aligned}$$

  (b)

$$\begin{aligned}
\Pr(\text{positive}) &= \Pr(\text{pregnant}) \times \Pr(\text{positive} \mid \text{pregnant}) + \Pr(\text{not pregnant}) \times \Pr(\text{positive} \mid \text{pregnant}) \\
&= 0.05 \times 0.98 + 0.95 \times (1 - 0.99) \\
&= 0.049 + 0.0095 = 0.0585
\end{aligned}$$

- **3.10 (pg 88)**

  (a)

$$\begin{aligned}
\Pr(\text{pregnant} \mid \text{positive}) &= \Pr(\text{positive and pregnant}) / \Pr(\text{positive}) \\
&= \Pr(\text{pregnant}) \times \Pr(\text{positive} \mid \text{pregnant}) / \Pr(\text{postive}) \\
&= (0.1 \times 0.98)/0.107 \\
&\approx 0.916
\end{aligned}$$

  (b)

$$\begin{aligned}
\Pr(\text{pregnant} \mid \text{positive}) &= \Pr(\text{pregnant}) \times \Pr(\text{positive} \mid \text{pregnant}) / \Pr(\text{postive}) \\
&= (0.05 \times 0.98)/0.0585 \\
&\approx 0.838
\end{aligned}$$

- **3.18 (pg 101)**

  (a) $\Pr(Y = 3) = 610/5000 = 0.122$

  (b) $\Pr(Y \geq 7) = (130 + 26 + 3 + 1)/5000 = 160/5000 = 0.032$

  (c) $\Pr(4 \leq Y \leq 6) = (1400 + 1760 + 750)/5000 = 0.782$

- **3.19 (pg 102)**

  (a) $\Pr(Y' = 3) = (610 \times 3)/22435 \approx 0.0816$

  (b) $\Pr(Y \geq 7) = (130 \cdot 7 + 26 \cdot 8 + 3 \cdot 9 + 1 \cdot 10)/22435 = 1155/22435 \approx 0.0515$

  (c) If one chooses *young* at random (rather than *broods*), one will tend to choose young that come from larger broods.

- **3.28 (pg 111)**

  (a) $\Pr(\text{all 20 will be cured}) = (0.9)^{20} \approx 0.122$
  In R: `dbinom(20, 20, 0.9)`

  (b) $\Pr(\text{all but one will be cured}) = 20 \times 0.9^{19} \times 0.1 \approx 0.270$
  In R: `dbinom(19, 20, 0.9)`

  (c) $\Pr(\text{exactly 18 will be cured}) = \binom{20}{2} \times 0.9^{18} \times 0.1^2 \approx 0.285$
  In R: `dbinom(18, 20, 0.9)`

  (d) This is the same as (c), since 90% of 20 is 18, so $\sim$0.285.

- **3.32 (pg 111)**

  (a) $\Pr(\text{all 8 cases will be detected}) = 0.7^8 \approx 0.058$
  In R: `dbinom(8, 8, 0.7)`

  (b) $\Pr(\text{only one case will be missed}) = \Pr(\text{exactly 7 cases will be detected}) = 8 \times 0.7^7 \times 0.3 \approx 0.198$
  In R: `dbinom(7, 8, 0.7)`

  (c) $\Pr(\text{two or more cases will be missed}) = 1 - \Pr(\text{8 cases detected}) - \Pr(\text{7 cases detected}) \approx 1 - 0.058 - 0.198 \approx 0.745$
  In R: `pbinom(6, 8, 0.7)`