Stat 371-003, Solutions to Homework #4

- **4.10 (pg 132)**

  $Y \sim$ normal(mean $= 176$, SD $= 30$)

  Let $Z = (Y - 176)/30$, so that $Z \sim$ normal(mean $= 0$, SD $= 1$)

  (a)
  $$\begin{aligned} \Pr(Y \geq 180) &= \Pr[Z \geq (180-176)/30] \\ &\approx \Pr(Z \geq 0.1333) \\ &= \Pr(Z \leq -0.1333) \\ &\approx \textbf{44.7\%}. \end{aligned}$$

  In R: `1-pnorm(180, 176, 30)` or `1-pnorm( (180-176)/30 )`

  (b)
  $$\begin{aligned} \Pr(180 < Y < 210) &= \Pr[(180-176)/30 < Z < (210-176)/30] \\ &\approx \Pr(0.1333 < Z < 1.1333) \\ &= \Pr(Z < 1.1333) - \Pr(Z < 0.1333) \\ &\approx 0.8715 - 0.5530 \\ &\approx \textbf{31.8\%}. \end{aligned}$$

  In R: `pnorm(210, 176, 30) - pnorm(180, 176, 30)`

- **4.11 (pg 132)**

  $Z = (Y - 176)/30$ has a standard normal distribution.

  Note that $Y = 30Z + 176$.

  (a) The 80th percentile of Z is approximately 0.8416. That is, $\Pr(Z < 0.8416) \approx 0.8$.
  And so $\Pr(Y < 30 \cdot 0.8416 + 176) \approx 0.8$.
  Thus $\Pr(Y < 201) \approx 0.8$.
  And so the 80th percentile of Y is approximately **201**.

  In R: `qnorm(0.8, 176, 30)`

  (b) The 20th percentile of Z is approximately -0.8416.
  And so $\Pr(Y < 176 - 30 \cdot 0.8416) \approx 0.2$.
  Thus $\Pr(Y < 151) \approx 0.2$.
  And so the 20th percentile of Y is approximately **151**.
  (One could also use symmetry: the 80th percentile is approx. 201; $201 - 176 = 25$, and so the 20th percentile must be approx. $176 - 25 = 151$.

  In R: `qnorm(0.2, 176, 30)`

- **4.34 (pg 146)**

  $Y \sim$ normal(mean $= 15.6$, SD $= 0.4$)

  Let $Z = (Y - 15.6)/0.4$, so that $Z \sim$ normal(mean $= 0$, SD $= 1$)

  (a)
  $$\begin{aligned}\Pr(Y > 15) &= \Pr[Z > (15 - 15.6)/0.4] \\ &\approx \Pr(Z > -1.5) \\ &= \Pr(Z < 1.5) \\ &\approx \textbf{93.3\%}.\end{aligned}$$

  In R: `1-pnorm(15, 15.6, 0.4)`

  (b)
  $$\begin{aligned}\Pr(Y > 16.5) &= \Pr[Z > (16.5 - 15.6)/0.4] \\ &\approx \Pr(Z > 2.25) \\ &= \Pr(Z < -2.25) \\ &\approx \textbf{1.2\%}.\end{aligned}$$

  In R: `1-pnorm(16.5, 15.6, 0.4)`

  (c)
  $$\begin{aligned}\Pr(15 < Y < 16.5) &= \Pr(Y > 15) - \Pr(Y > 16.5) \\ &= (a) - (b) \\ &= 93.3\% - 1.2\% \\ &= \textbf{92.1\%}.\end{aligned}$$

  In R: `pnorm(16.5, 15.6, 0.4) - pnorm(15, 15.6, 0.4)`

  (d)
  $$\begin{aligned}\Pr(15 < Y < 15.5) &= \Pr[(15 - 15.6)/0.4 < Z < (15.5 - 15.6)/0.4] \\ &= \Pr(-1.5 < Z < -0.25) \\ &= \Pr(Z < -0.25) - \Pr(Z < -1.5) \\ &\approx 0.4013 - 0.0668 \\ &\approx \textbf{33.4\%}.\end{aligned}$$

  In R: `pnorm(15.5, 15.6, 0.4) - pnorm(15, 15.6, 0.4)`

- **5.4 (pg 156)**

  Let X = the number of patients (out of 15) who respond to treatment, and let $\hat{p} = X/15$.

  Note that $X \sim$ binomial$(n = 15, p = 0.2)$.

  (a)
  $$\begin{aligned} \Pr(\hat{p} = 0.2) &= \Pr(X = 0.2 \times 15) \\ &= \Pr(X = 3) \\ &= \binom{15}{3}(0.2)^3(0.8)^{12} \\ &\approx \mathbf{25\%}. \end{aligned}$$

  In R: `dbinom(3, 15, 0.2)`

  (b)
  $$\begin{aligned} \Pr(\hat{p} = 0) &= \Pr(X = 0) \\ &= (0.8)^{15} \\ &\approx \mathbf{3.5\%}. \end{aligned}$$

  In R: `dbinom(0, 15, 0.2)`

- **5.15 (pg 164)**

  (a) Let $Y \sim$ normal(mean $= 176$, SD $= 30$), and let $Z = (Y - 176)/30$, so that $Z \sim$ normal(mean $= 0$, SD $= 1$).

  $$\begin{aligned} \Pr(166 < Y < 186) &= \Pr[(166 - 176)/30 < Z < (186 - 176)/30] \\ &\approx \Pr(-0.3333 < Z < 0.3333) \\ &= \Pr(Z < 0.3333) - \Pr(Z < -0.3333) \\ &\approx 0.6306 - 0.3694 \\ &\approx \mathbf{26.1\%}. \end{aligned}$$

  In R: `pnorm(186, 176, 30) - pnorm(166, 176, 30)`

  (b) Let $\bar{Y}$ denote the sample average, so that $\bar{Y} \sim$ normal(mean $= 176$, SD $= 30/\sqrt{9} = 10$). Let $\bar{Z} = (\bar{Y} - 176)/10$ so that $Z \sim$ normal$(0, 1)$.

  $$\begin{aligned} \Pr(166 < \bar{Y} < 186) &= \Pr[(166 - 176)/10 < \bar{Z} < (186 - 176)/10] \\ &= \Pr(-1 < \bar{Z} < 1) \\ &\approx \mathbf{68\%}. \end{aligned}$$

  In R: `pnorm(186, 176, 10) - pnorm(166, 176, 10)`

  (c) This is the same as (b).

- **5.23 (pg 165)**

  (a) Let $Y \sim$ normal(mean $= 35, \text{SD} = 4$), and let $Z = (Y - 35)/4$, so that $Z \sim$ normal(mean $= 0, \text{SD} = 1$).

$$\begin{aligned}\Pr(Y > 40) &= \Pr[Z > (40 - 35)/4] \\ &= \Pr(Z > 1.25) \\ &= \Pr(Z < -1.25) \\ &\approx \mathbf{10.6\%}.\end{aligned}$$

  In R: `1 - pnorm(40, 35, 4)`

  (b) Let $\bar{Y}$ denote the average of the 3 measurements, so that $\bar{Y} \sim$ normal(mean $= 35, \text{SD} = 4/\sqrt{3} \approx 2.309$). Let $\bar{Z} = (\bar{Y} - 35)/(4/\sqrt{3})$ so that $Z \sim$ normal$(0, 1)$.

$$\begin{aligned}\Pr(\bar{Y} > 40) &\approx \Pr[\bar{Z} > (40 - 35)/2.309] \\ &\approx \Pr(Z > 2.165) \\ &= \Pr(Z < -2.165) \\ &\approx \mathbf{1.5\%}.\end{aligned}$$

  In R: `1-pnorm(40, 35, 4/sqrt(3))`