Stat 371-003, Solutions to Homework #7

1. **9.17 (pg 370)**

   Here we are doing a one-sided test (as the alternative hypothesis is "environmental enrichment tends to increase the relative size of the cortex". And so p-value is the chance of observing 10 or more pairs for which the "enriched" rat has larger relative cortex weight than the "impoverished" rat, if the treatment has no effect on the outcome.

   This is the same a the chance of getting 10 or more heads in 12 tosses of a fair coin. Let X = number of heads.

$$\begin{aligned}\Pr(X \geq 10) &= \Pr(X = 10) + \Pr(X = 11) + \Pr(X = 12) \\ &= \binom{12}{10}(0.5)^{12} + \binom{12}{11}(0.5)^{12} + (0.5)^{12} \\ &= 66 \times (0.5)^{12} + 12 \times (0.5)^{12} + (0.5)^{12} \\ &\approx 0.019 = \mathbf{1.9\%}\end{aligned}$$

   In R, we could type `1-pbinom(9,12,0.5)` or `binom.test(10,12, 0.5, alternative="greater")`.

   Since the p-value is small, we reject the null hypothesis and conclude that enrichment does affect relative cortex weight.

2. **9.30 (pgs 375-376)**

   The absolute values of the differences are 5, 7, 28, 47, 80, 7, 8 and 20.

   The ranks of the absolute differences are 1, 2.5, 6, 7, 8, 2.5, 4, and 5. (The two 7's would have ranks 2 and 3; we assign the average rank to each.)

   The signed ranks are –1, 2.5, –6, –7, –8, 2.5, 4, and –5.

   The sum of the negative ranks is 1+6+7+8+5 = 27.

   The sum of the positive ranks is 2.5+2.5+4=9.

   Looking up **27** in table 8 for $n = 8$, we find that the **p-value is** $>$ **0.2**, and so we fail to reject the null hypothesis. There is insufficient evidence to conclude that the treatment has an effect.

   In R, we could type:

```
x <- c(-5, 7, -28, -47, -80, 7, 8, -20)
wilcox.test(x)
```

   We get a p-value of 23%.

3. **7.80 (pgs 296-297)**

   Using the procedure described in the book, we first calculate, for each value in the treatment group, the number of values in the control group that are smaller. This gives the values 4, 5, 6, 6, 8, 8, 8, 8. The sum is $K_1 = \mathbf{53}$.

   We do the same for the control group, giving the numbers 0, 0, 0, 0, 1, 2, 4, 4. The sum is $K_2 = \mathbf{11}$.

   The larger of these two values is $U_s = \mathbf{53}$. Looking this up in Table 6 (for $n = n' = 8$) we find that the p-value is between 2% and 5%, and so we reject the null hypothesis and conclude that hypnosis does affect total ventilation.

   In R, we could type:

   ```
   x <- c(5.32, 5.60, 5.74, 6.06, 6.32, 6.34, 6.79, 7.18)
   y <- c(4.50, 4.78, 4.79, 4.86, 5.41, 5.70, 6.08, 6.21)
   wilcox.test(x,y)
   ```

   We get a p-value of 2.8%.

4. **7.79 (pg 296)**

   We load the data into R as follows:

   ```
   dat <- read.csv("http://www.biostat.wisc.edu/%7Ekbroman/teaching/stat371/data_7-79.csv")
   x <- dat$dopamine[dat$treatment=="toluene"]
   y <- dat$dopamine[dat$treatment=="control"]
   ```

   (a) t **test**

   `t.test(x,y)` gives a p-value of **0.022**.

   (b) Rank-sum test

   `wilcox.test(x,y)` gives a p-value of **0.026**.

   (c) Permutation test

   To calculate the p-value from the permutation test, we type:

   ```
   source("http://www.biostat.wisc.edu/%7Ekbroman/teaching/stat371/permfunc.R")
   perm.test(x, y, n.perm=NULL, pval=TRUE)
   ```

   This gives a p-value of **0.024**.

5. **6.48 (pg 213)**

   (a) To calculate the 95% confidence interval by the normal approximation, we first calculate the estimated probability and its estimated standard error: $\hat{p} = 103/1438 \approx 0.0716$,

   $$\hat{\text{SE}}(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \approx 0.0068$$

   The 95% confidence interval is then 0.0716 $\pm$ 1.96 $\times$ 0.0068 = 0.0716 $\pm$ 0.0133, or the interval **(5.8%, 8.5%)**.

(b) To get the exact 95% CI using R, we type `binom.text(103, 1438)`. We get the interval **(5.9%, 8.6%)**.