Problem Set 1 Special topics in genetics and genomics (140.668)
Due 15 Nov 2004

1. Prove the claim from the proof of Mather's formula:

   If $m|n \sim$ Binomial$(n, 1/2)$, then

$$\Pr(m \text{ is odd}|n) = \begin{cases} 0 & \text{if} \quad n = 0 \\ 1/2 & \text{if} \quad n \geq 1 \end{cases}$$

2. Use Mather's formula to derive the map function for the count-location model when the distribution of the number of chiasmata on the four-strand bundle is $\boldsymbol{p} = (p_0, p_1, p_2, \ldots)$. (Let $L = \sum_i i p_i / 2$ denote the genetic length of the chromosome.) Show that in the case $p_n = \exp(-2L)(2L)^n/n!$, one obtains the Haldane map function.

3. Show that if the locations of chiasmata on the four-strand bundle follow a Poisson process, then under no chromatid interference, the locations of crossovers on a random meiotic product also follow a Poisson process.

4. Find a combination of chromatid and chiasma interference for which the locations of crossovers on a random meiotic product follow a stationary Poisson process. What do you conclude?

   Hint: Use the facts that if $X_1, X_2, \ldots, X_n$ are independent with $X_i \sim$ Gamma(shape = $\nu_i$, rate = $\lambda$), then $\sum X_i \sim$ Gamma($\sum \nu_i$, $\lambda$), and that Gamma($\nu = 1$, $\lambda$) = Exponential($\lambda$).