**Aligned Rank Tests: An Application to Clinical Trials**

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**R Estimation w/ General Scores**

Jurečková (1971) and Jaeckel (1972) developed rank (R) estimation for general model (2). The monograph by Hettmansperger and McKean (2011) provides a recent review of rank (R) estimation for linear models.

The objective function is Jacek's dispersion function which is defined as

\[ \|v\|_\varphi = \sum_{i=1}^{n} |a(R(y_i))v_i| \]

where \( R \) denotes rank and \( a(R(y_i)) = 0 \) for \( t = 1, \ldots, n \) are the scores. The scores are nondecreasing and defined as

\[ a(t) = \varphi \left( \frac{t}{n+1} \right) \]

where \( \varphi \) is nondecreasing on \((0,1)\) and standardized so that \( \int_0^1 \varphi(u) \, du = 0 \) and \( \int_0^1 \varphi(u)^2 \, du = 1 \).

Example \( \varphi \) or score functions are displayed in Figure 2.

**Rank-Based (R) Estimation**

The R estimator of \( b \) in (2) is the minimizer of Jacek's dispersion function (3).

\[ b_\varphi = \text{Argmin} \{ |y - Xb_\varphi| \} = \text{Argmin} \left\{ \sum_{i=1}^{n} |a(R(y_i) - x_i^Tb)|/|y_i - x_i^Tb| \right\}. \]

Note: Under regularity conditions (see Hettmansperger and McKean, 2011) \( b_\varphi \) is consistent & asymptotically normal.

R estimates may be obtained in the R package Rfit (Kloke and McKean, 2012).

**Gradient Process**

The negative of the gradient of Jacek's dispersion function is

\[ S(b) = X^T a(R(y - Xb)). \]

**Simulation Studies**

Simulation studies were conducted by obtaining a simple random sample from the complete LDL dataset (Figure 1) for each treatment arm. Power, to detect the apparent treatment effect, and level were considered. Empirical level was calculated by randomizing treatment assignment for the sample. Simulation size was 10000. Nominal \( \alpha = 0.005 \). Results are presented in Tables 1-2.

An ROC curve (Figure 3) was calculated by sampling from a distribution with half the overall effect size. That is the treatment (high dose) arm for the sample was obtained by sampling equally from the high and low dose arms in the complete LDL data (Figure 1). The sample size was \( n = 16 \) per group.

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**References**


